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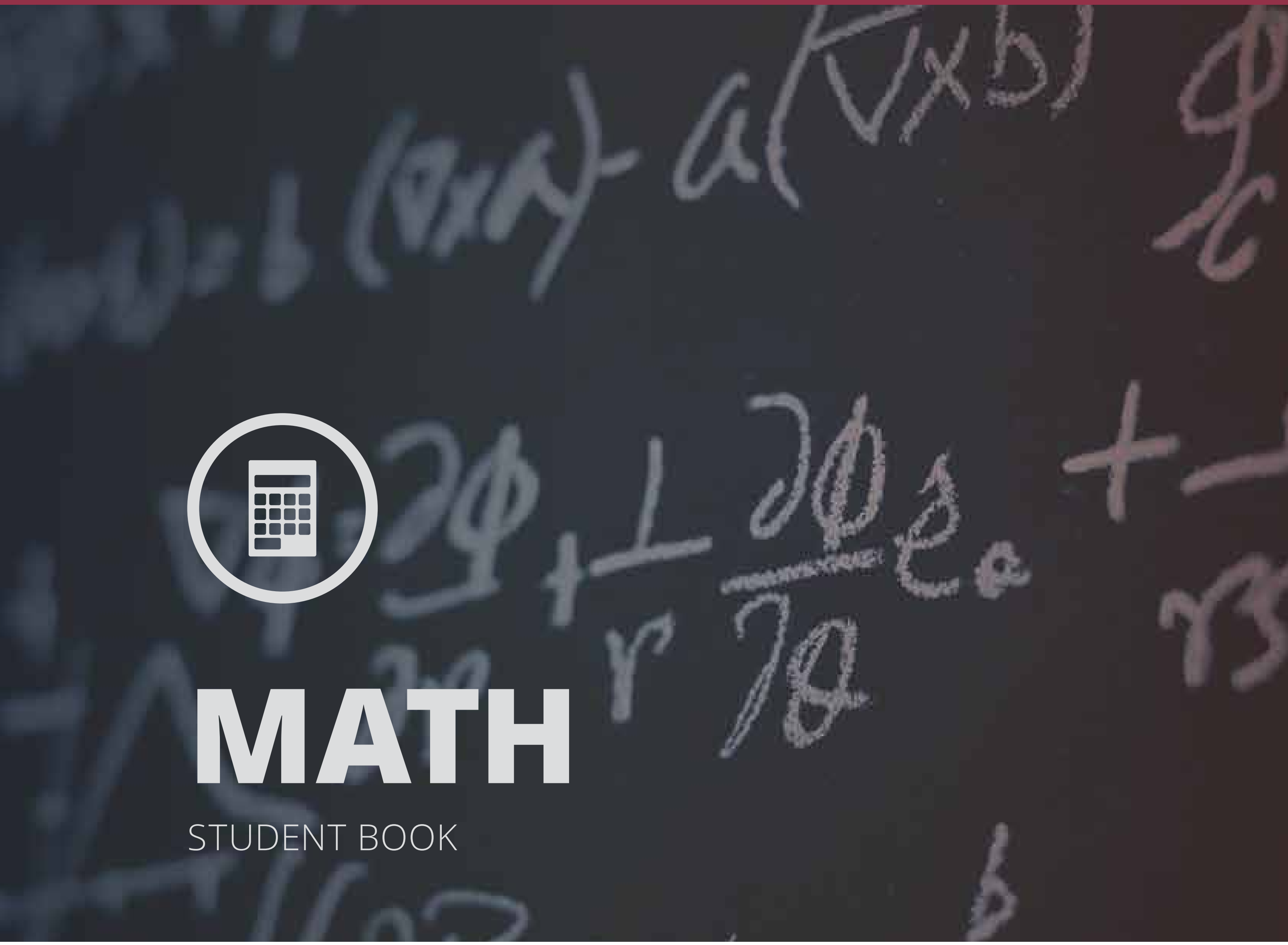

MATH

STUDENT BOOK

11th Grade | Unit 7

Alpha Omega
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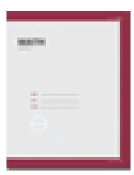
MATH 1107

QUADRATIC RELATIONS AND SYSTEMS



LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Quadratic Relations and Systems

Introduction

Millions of physical scientists, managerial specialists, social scientists, engineers, and mathematicians have studied and used the quadratic relations known as conic sections. In this LIFEPAC, you will learn to describe by equation and graph the circle, ellipse, parabola, and hyperbola. You will also examine some of their applications. The study will begin with the formula for the distance between two points, which is basic to understanding these quadratic relations.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC®. When you have finished this LIFEPAC, you should be able to:

1. Use the distance formula to calculate the distance between two points in a coordinate plane.
2. Define this circle and ellipse and use the distance formula to derive their equations.
3. Graph the circle and ellipse and describe essential elements from their equations.
4. Write the equations of the circle and ellipse from the graphs and from descriptions of the essential elements.
5. Define the parabola and hyperbola and use the distance formula to derive their equations.
6. Graph the parabola and hyperbola and describe essential elements from their equations.
7. Write the equations of the parabola and hyperbola from the graphs and from descriptions of the essential elements.
8. Identify each conic section from its equation.
9. Solve and graph systems of first- and second-degree equations.
10. Solve and graph second-degree inequalities.
11. Solve application problems involving conic sections.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. DISTANCE FORMULA AND CONIC SECTIONS

The distance formula is needed in deriving the general equations for the four conic sections we shall study in this LIFEPAC. The use of the distance formula will become clearer to you as you study the circle and ellipse.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

1. Use the distance formula to calculate the distance between two points in a coordinate plane.
2. Define the circle and ellipse and use the distance formula to derive their equations.
3. Graph the circle and ellipse and describe essential elements from their equations.
4. Write the equations of the circle and ellipse from the graphs and from descriptions of the essential elements.

DISTANCE FORMULA

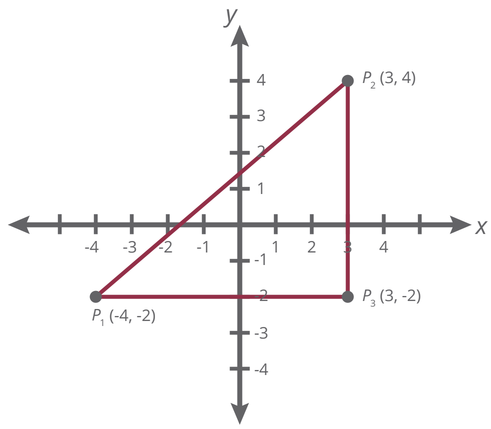
Suppose that P_2 (3, 4) and P_1 (-4, -2) are two points in the coordinate plane.

You know that some specific distance exists between P_1 and P_2. If we locate an extra point P_3 making a right triangle, we can write by the Pythagorean Theorem:

The distance between P_1 and P_2, denoted by $|P_1P_2|$ or d, =

$$\sqrt{(\text{distance between } P_1 \text{ and } P_3)^2 + (\text{distance between } P_3 \text{ and } P_2)^2}$$

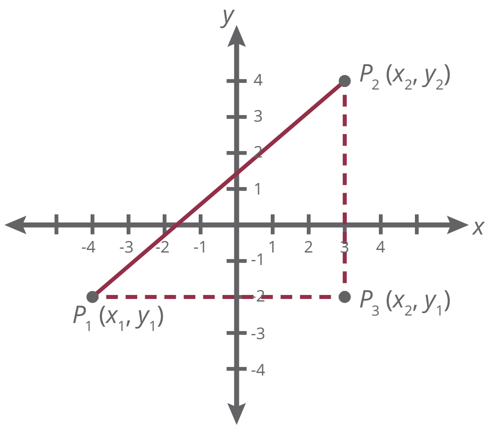
$$= \sqrt{[3-(-4)]^2 + [4-(-2)]^2} = \sqrt{(7)^2 + (6)^2} = \sqrt{85} \cong 9.2$$



Thinking along the same lines, we can obtain a more useful and general formula for the distance between P_1 (x_1, y_1) and P_2 (x_2, y_2).

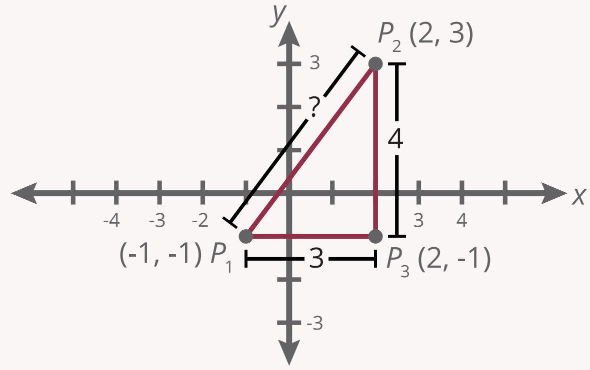
DISTANCE FORMULA

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



See if the results from the distance formula are reasonable by applying the formula to the familiar 3-4-5 right triangle in the model.

Model 1:

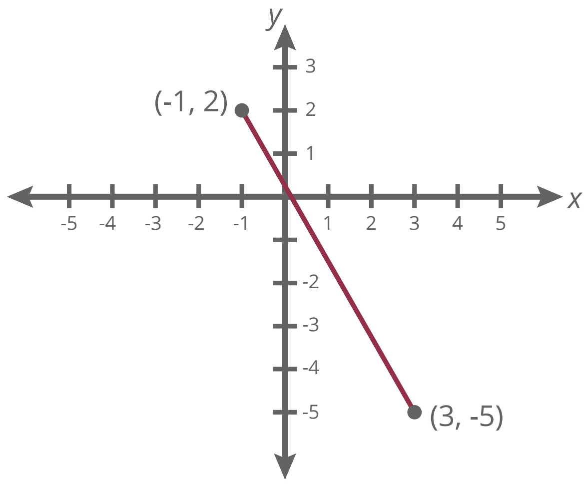


Calculate: $|P_1P_2| = \sqrt{[2 - (-1)]^2 + [3 - (-1)]^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

Should $|P_1P_2|$ be 5? Yes.

Model 2: Graph and find the distance between (-1, 2) and (3, -5).

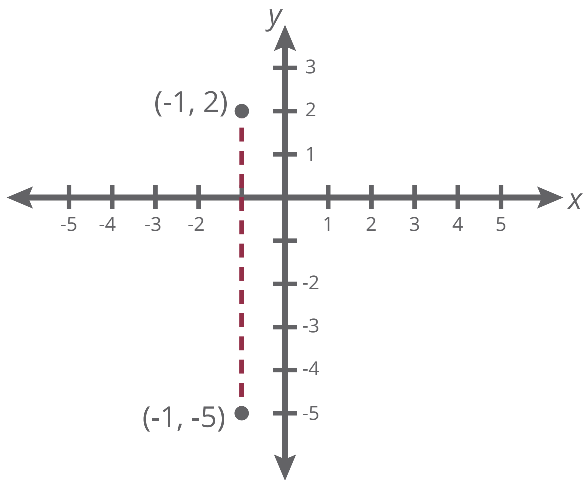
$$d = \sqrt{[3 - (-1)]^2 + (-5 - 2)^2}$$
$$= \sqrt{16 + 49} = \sqrt{65} \cong 8.1$$



Model 3: Graph and find the distance between (-1, 2) and (-1, -5).

We could use the distance formula; however, in cases of horizontal or vertical lines, use the absolute value of the difference between the *x*-coordinates or the *y*-coordinates:

$$d = |2 - (-5)| = 7$$



when helpful or necessary.

1.1 (1, 2) and (4, 3)

1.2 (0, 6) and (2, 3)

1.3 (0, 0) and (6, -8)

1.4 (-3, -2) and (-1, -2)

1.5 (0, 0) and (4, 0)

1.6 (0, 0) and (0, 8)

1.7 (1, 5) and (1, -4)

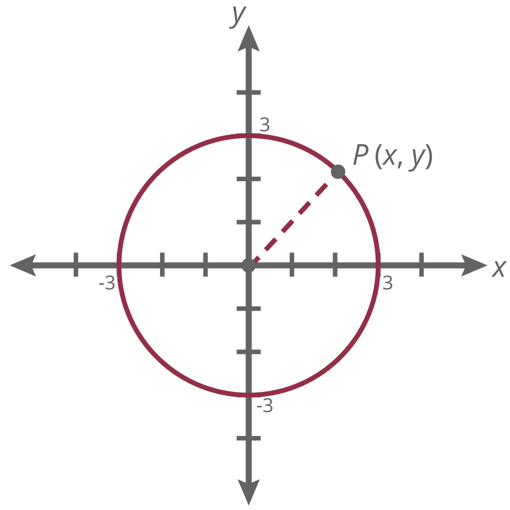
Complete these activities.

1.8 From memory, draw a right triangle with P_1 (x_1, y_1) and P_2 (x_2, y_2) and write the distance formula.

1.9 Use the distance formula to show that the points A_1 (0, 4), A_2 (2, 8), and A_3 (-1, 2) are on a straight line. (*Hint:* Graph the points and show that $|A_3A_2| = |A_2A_1| + |A_3A_1|$.)

CIRCLE

The circle in a coordinate plane can be thought of as a set of points *P*, with coordinates (x, y), that are all a constant radius (distance) from the center.



A circle is shown with center at (0, 0) and radius of 3. We can think of this circle as the set of all points *P*, with coordinates (x, y), that are a distance of 3 units from the center (0, 0). Writing these conditions with the distance formula, we have the following equation of the circle:

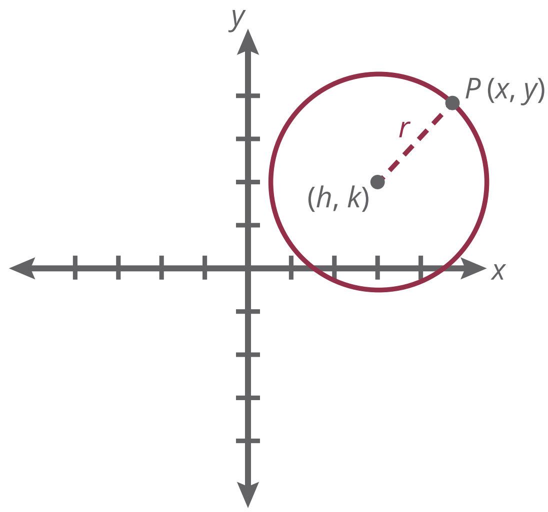
$$\sqrt{(x-0)^2+(y-0)^2} = 3$$

Squaring both sides, we have an equation of the circle of radius 3 and center (0, 0):

$$(x-0)^2+(y-0)^2 = 9$$

Thinking along these lines about a more general circle with center (h, k) and radius r, we have by the distance formula the equation of the circle that is the set of all points *P*, with coordinates (x, y), that are a distance of r units from the center (h, k):

$$\sqrt{(x-h)^2+(y-k)^2} = r$$



This equation yields even more information about the circle, as shown.

Circle: $(x-h)^2+(y-k)^2 = r^2$

Center: (h, k)

Radius = r

Consider the circle $(x-3)^2+(y+6)^2 = 12$.
By expanding the expressions we have

$$x^2+y^2-6x+12y+33 = 0,$$

suggesting that an equation of the form

$$Ax^2+Cy^2+Dx+Ey+F = 0$$

might be the equation of a circle.

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