



# MATH

STUDENT BOOK

▶ **9th Grade** | Unit 9

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# Math 909

## Systems

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**LIFEPAC Test is located in the center of the booklet.** Please remove before starting the unit.

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# Systems

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## INTRODUCTION

In this LIFEPAK<sup>®</sup>, you will continue your study in *algebra* by learning to find any common solutions to groups of open sentences called *systems*—first graphically using the techniques of the preceding LIFEPAK, then algebraically using several different methods. Finally, you will see how systems can be set up to solve verbal problems of various types.

## Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAK. When you have finished this LIFEPAK, you should be able to:

1. Identify the equations of systems as consistent, equivalent, or inconsistent.
2. Solve systems of linear equations and inequalities by graphing
3. Solve systems of linear equations by the methods of opposite coefficients, comparison, substitution, and determinants.
4. Solve problems using systems of linear equations.

# 1. GRAPHICAL SOLUTIONS

You are already familiar with the procedures for drawing the graph of a linear equation or a linear inequality on the real-number plane. In this section, we shall graph more than one such open sentence

on the same grid and then determine whether ordered pairs exist that satisfy the system. You will need to draw your graphs as accurately as possible.

## OBJECTIVES

Review these objectives. When you have completed this section, you should be able to:

1. Identify the equations of systems as consistent, equivalent, or inconsistent.
2. Solve systems of linear equations and inequalities by graphing.

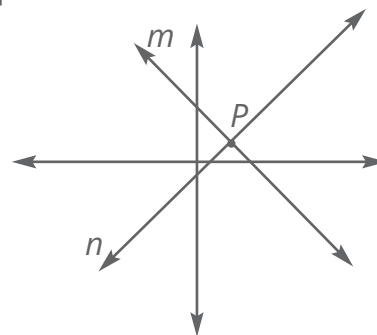
## EQUATIONS

We shall begin by looking at systems made up of two-variable linear equations. You need to learn several terms and then learn the procedures for solving these systems graphically.

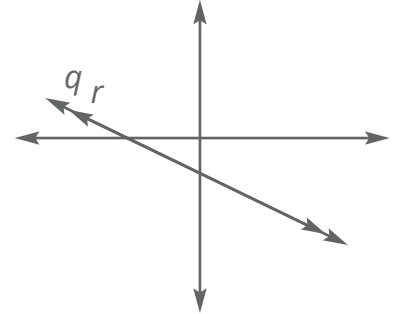
### TERMINOLOGY

A system of two linear equations is classified by the number of ordered pairs that satisfy both equations. Since the graph of each linear equation is a line, three possible situations can occur. These cases are shown in the following models.

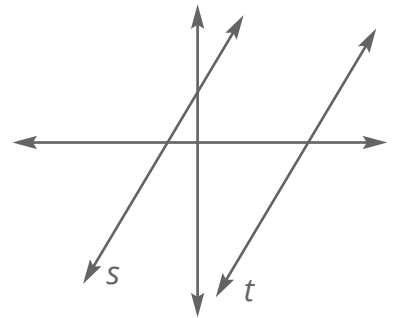
**Model 1:** Lines  $m$  and  $n$  intersect at one common point  $P$ .



**Model 2:** Lines  $q$  and  $r$  *coincide*, having all (infinitely many) common points.



**Model 3:** Lines  $s$  and  $t$  are *parallel*, having no common point.

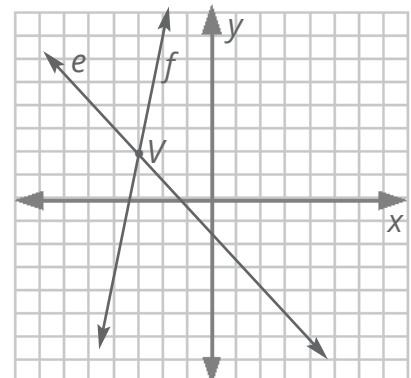


The equations of each of these three systems are identified as *consistent*, *equivalent*, and *inconsistent*, respectively. The set of the ordered pair(s) corresponding to any common point(s) is written as the *solution set* for each system.

### VOCABULARY

**consistent**—In a system, equations having a common solution are consistent.

**Model:** The equations for lines  $e$  and  $f$  are consistent. The solution set for this system is  $\{(-3, 2)\}$ , since the common point is  $V$ .

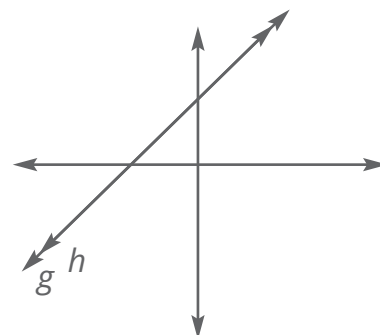


**VOCABULARY**

**equivalent**—In a system, equations having all common solutions are equivalent.

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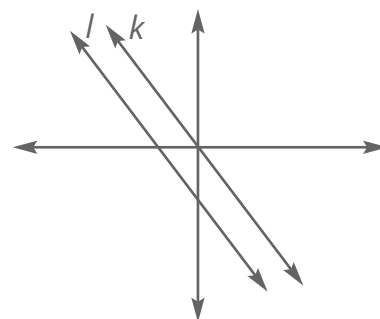
**Model:** The equations for lines  $g$  and  $h$  are equivalent. The solution set for this system is the infinite set of ordered pairs for all points on the line.

**VOCABULARY**

**inconsistent**—In a system, equations having no common solutions are inconsistent.

---

**Model:** The equations for lines  $k$  and  $l$  are inconsistent. The solution set for this system is  $\emptyset$  (the empty set).

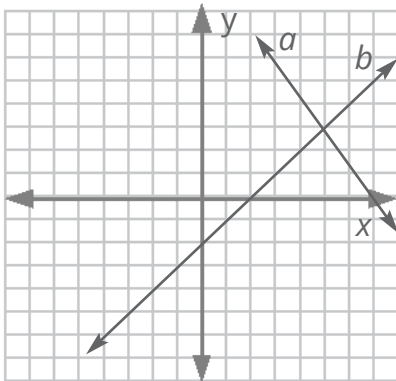




For each of the following pairs of lines, write a. the type of equations (consistent, equivalent, or inconsistent) and b. the solution set of each system.

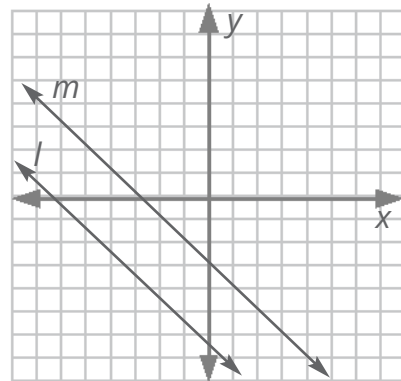
1.1 a. \_\_\_\_\_

b. \_\_\_\_\_



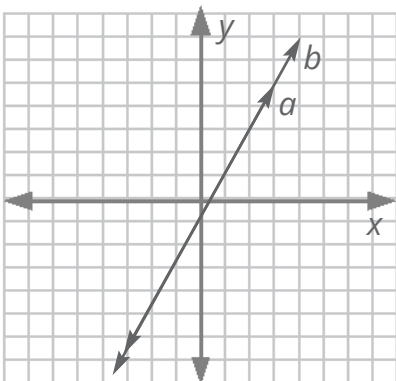
1.2 a. \_\_\_\_\_

b. \_\_\_\_\_



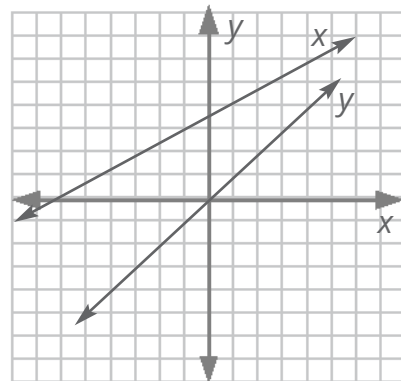
1.3 a. \_\_\_\_\_

b. \_\_\_\_\_



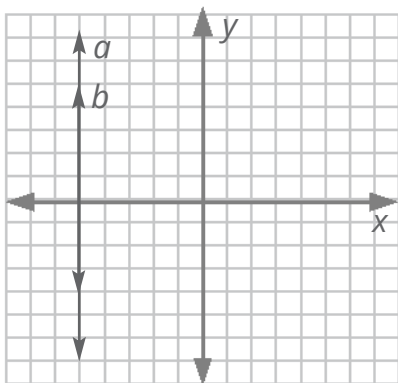
1.4 a. \_\_\_\_\_

b. \_\_\_\_\_

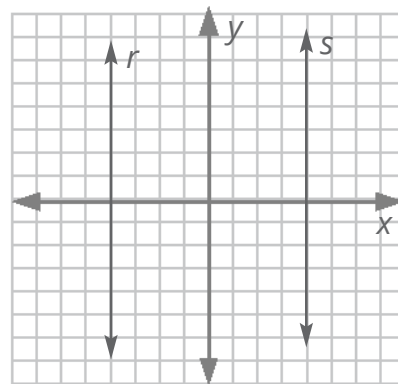




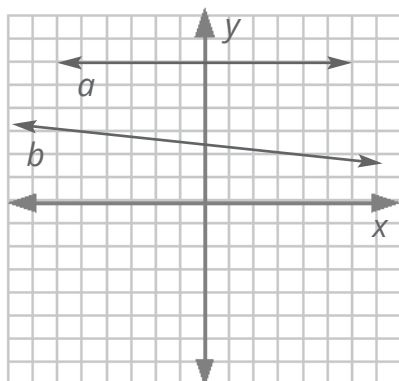
1.5 a. \_\_\_\_\_  
 b. \_\_\_\_\_



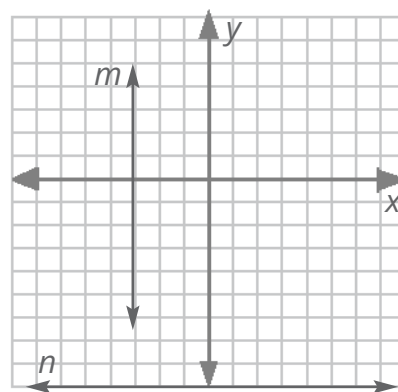
1.6 a. \_\_\_\_\_  
 b. \_\_\_\_\_



1.7 a. \_\_\_\_\_  
 b. \_\_\_\_\_

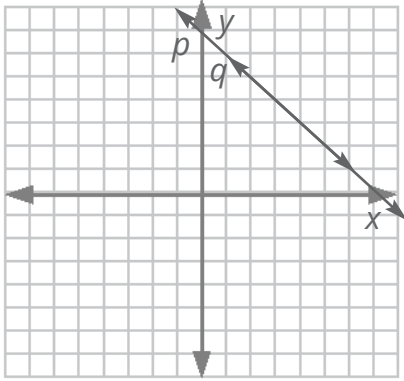


1.8 a. \_\_\_\_\_  
 b. \_\_\_\_\_



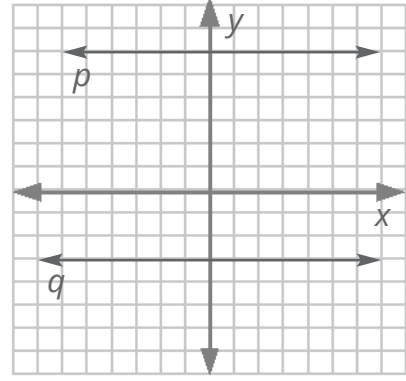
1.9 a. \_\_\_\_\_

b. \_\_\_\_\_



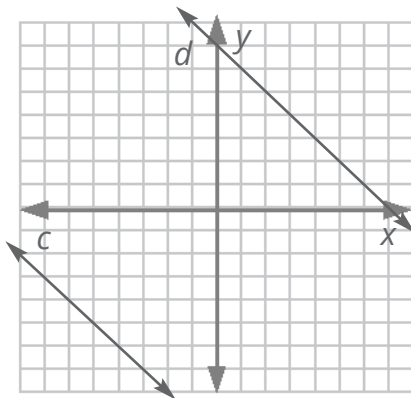
1.10 a. \_\_\_\_\_

b. \_\_\_\_\_



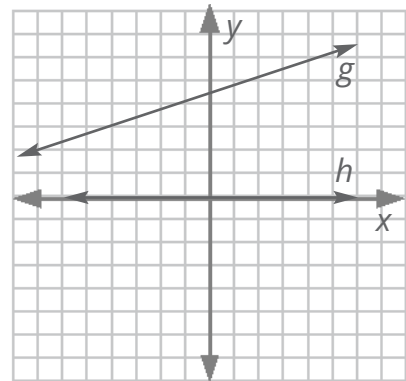
1.11 a. \_\_\_\_\_

b. \_\_\_\_\_



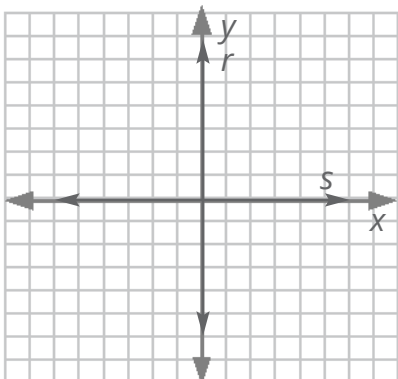
1.12 a. \_\_\_\_\_

b. \_\_\_\_\_



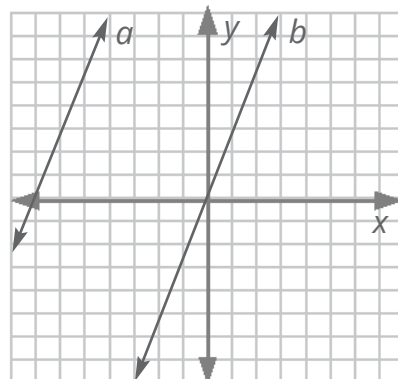
1.13 a. \_\_\_\_\_

b. \_\_\_\_\_



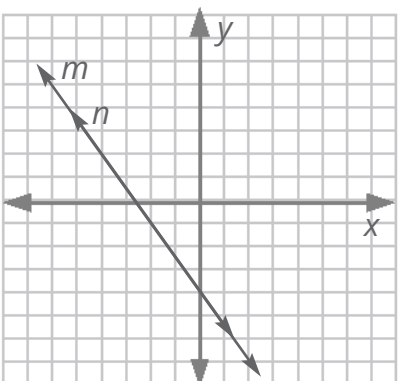
1.14 a. \_\_\_\_\_

b. \_\_\_\_\_



1.15 a. \_\_\_\_\_

b. \_\_\_\_\_



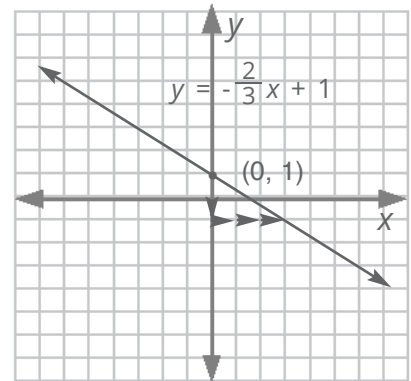
**GRAPHS**

Now we shall draw the graphs of linear equations to solve a system. This section will be a review of the techniques you learned in Mathematics LIFE PAC 908, except that you will be graphing two lines on the same number-plane grid.

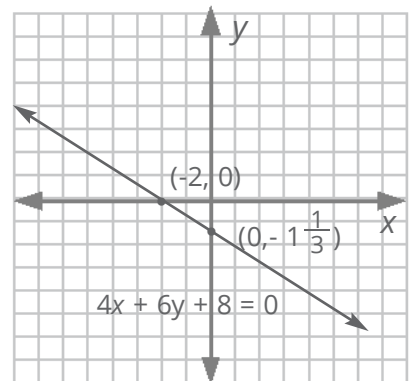
**Model 1:** Graph and describe the system

$$\begin{cases} y = -\frac{2}{3}x + 1 \\ 4x + 6y + 8 = 0. \end{cases}$$

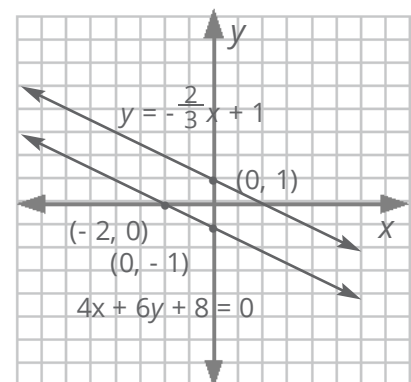
**Solution:** Step 1. The first equation,  $y = -\frac{2}{3}x + 1$ , is in slope,  $y$ -intercept form. Draw the line through  $(0, 1)$  and with a slope of  $-\frac{2}{3}$ .



Step 2. Use the intercepts method to graph the second equation,  $4x + 6y + 8 = 0$ . (When  $y = 0$ , the  $x$ -intercept is  $-2$ ; and when  $x = 0$ , the  $y$ -intercept is  $-\frac{4}{3}$  or  $-1.\bar{3}$ .)



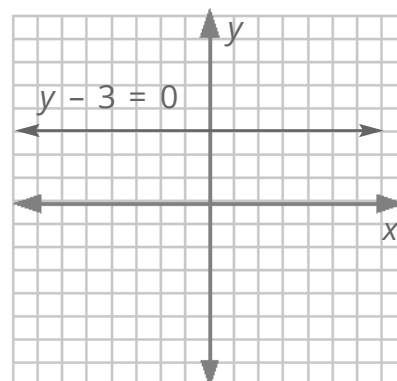
Step 3. Since the lines are parallel, the equations are inconsistent and the solution set of this system is  $\emptyset$ .



**Model 2:** Graph and describe the system

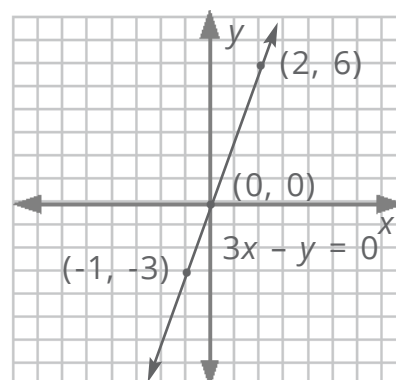
$$\begin{cases} y - 3 = 0 \\ 3x - y = 0. \end{cases}$$

**Solution:** Step 1. The first equation,  $y - 3 = 0$ , (or  $y = 3$ ) gives the horizontal line 3 units above the  $x$ -axis.



Step 2. You can set up a table of values to find points on the line for the second equation,  $3x - y = 0$  (or  $3x = y$ ).

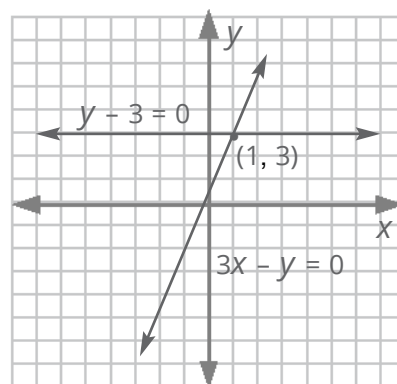
$x$	2	-1	0
$y$	6	-3	0



Step 3. Since the lines intersect in one point, the equations are consistent and the solution set of this system is  $\{(1, 3)\}$ .

**Checks:**

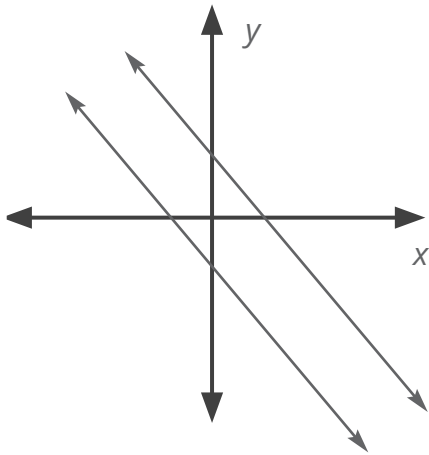
$y - 3 = 0$	$3x - y = 0$
$3 - 3 \stackrel{?}{=} 0$	$3 \cdot 1 - 3 \stackrel{?}{=} 0$
$0 = 0 \checkmark$	$3 - 3 \stackrel{?}{=} 0$
	$0 = 0 \checkmark$



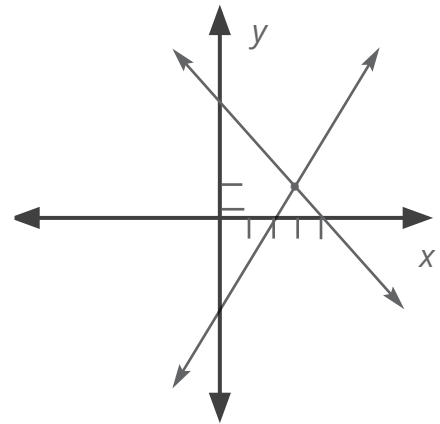
# SELF TEST 1

For each system of lines, a. write the type and b. find the solution set (each answer, 2 points).

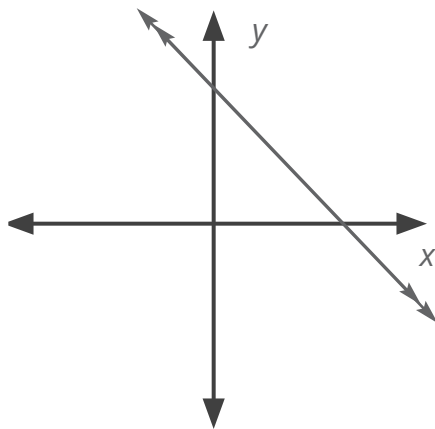
1.01 a. \_\_\_\_\_  
b. \_\_\_\_\_



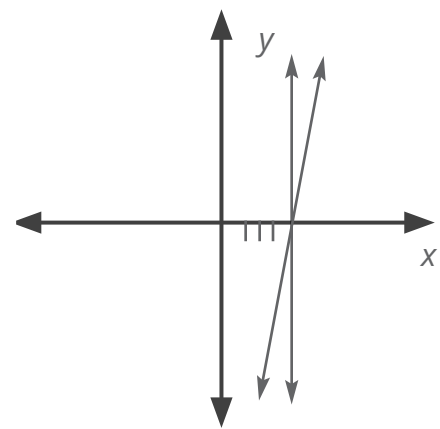
1.02 a. \_\_\_\_\_  
b. \_\_\_\_\_



1.03 a. \_\_\_\_\_  
b. \_\_\_\_\_

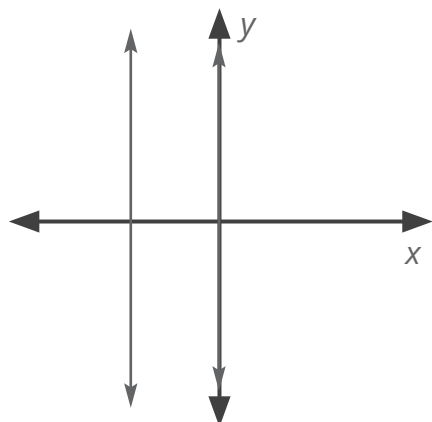


1.04 a. \_\_\_\_\_  
b. \_\_\_\_\_.



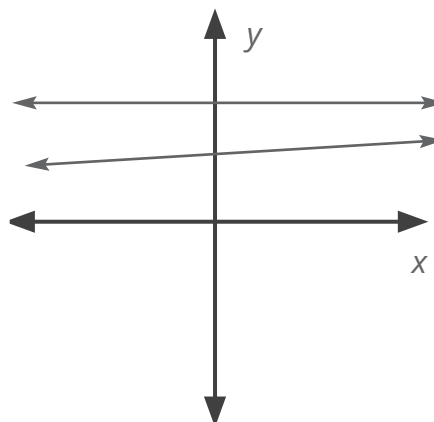
1.05 a. \_\_\_\_\_

b. \_\_\_\_\_



1.06 a. \_\_\_\_\_

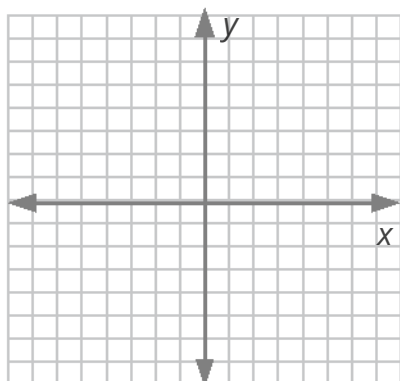
b. \_\_\_\_\_



Find the solution set by graphing (each graph, 4 points).

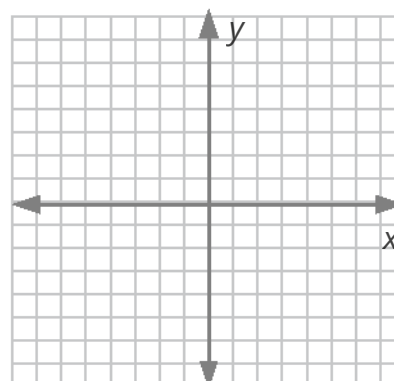
1.07

$$\begin{cases} x - y = 4 \\ x + y = 2 \end{cases}$$



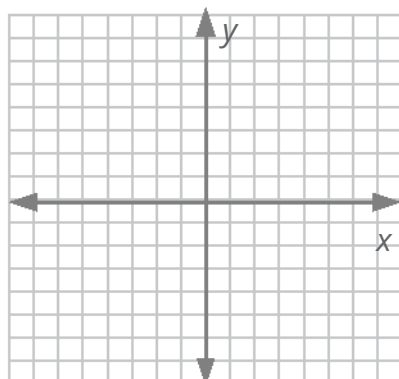
1.08

$$\begin{cases} y = 2x \\ x + 2y = 2 \end{cases}$$



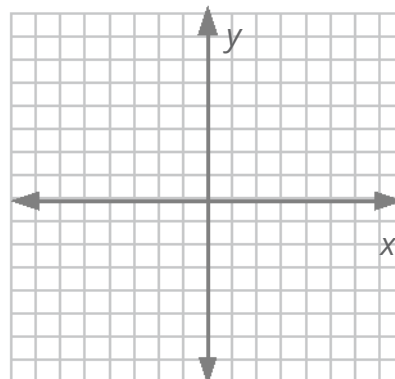
1.09

$$\begin{cases} 2x + 3y - 6 = 0 \\ x - y = 0 \end{cases}$$



1.010

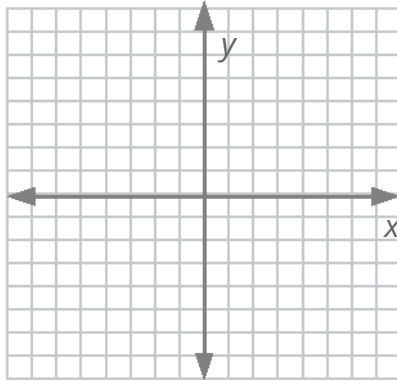
$$\begin{cases} y = -3 \\ x - y = 8 \end{cases}$$



Graph the solution for each of the following pairs of inequalities. Show only the final result for each system (each graph, 4 points).

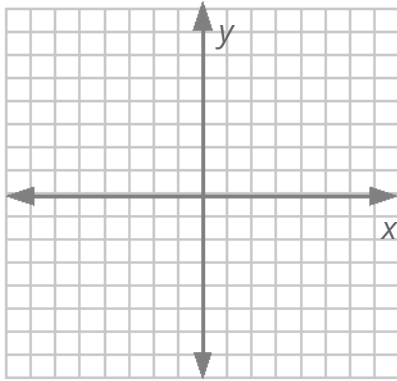
1.011

$$\begin{cases} y \leq x \\ x + y \geq 1 \end{cases}$$



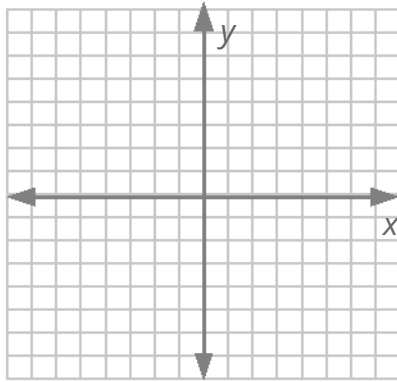
1.012

$$\begin{cases} x + y > 3 \\ x + y < -4 \end{cases}$$



1.013

$$\begin{cases} 2x + y < 1 \\ y \geq -4 - 2x \end{cases}$$



<div style="border: 1px solid black; padding: 5px; display: inline-block;">                 42  <hr style="width: 50%; margin: 0;"/>                 52             </div>	<b>SCORE</b> _____	<b>TEACHER</b> _____	initials _____ date _____
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