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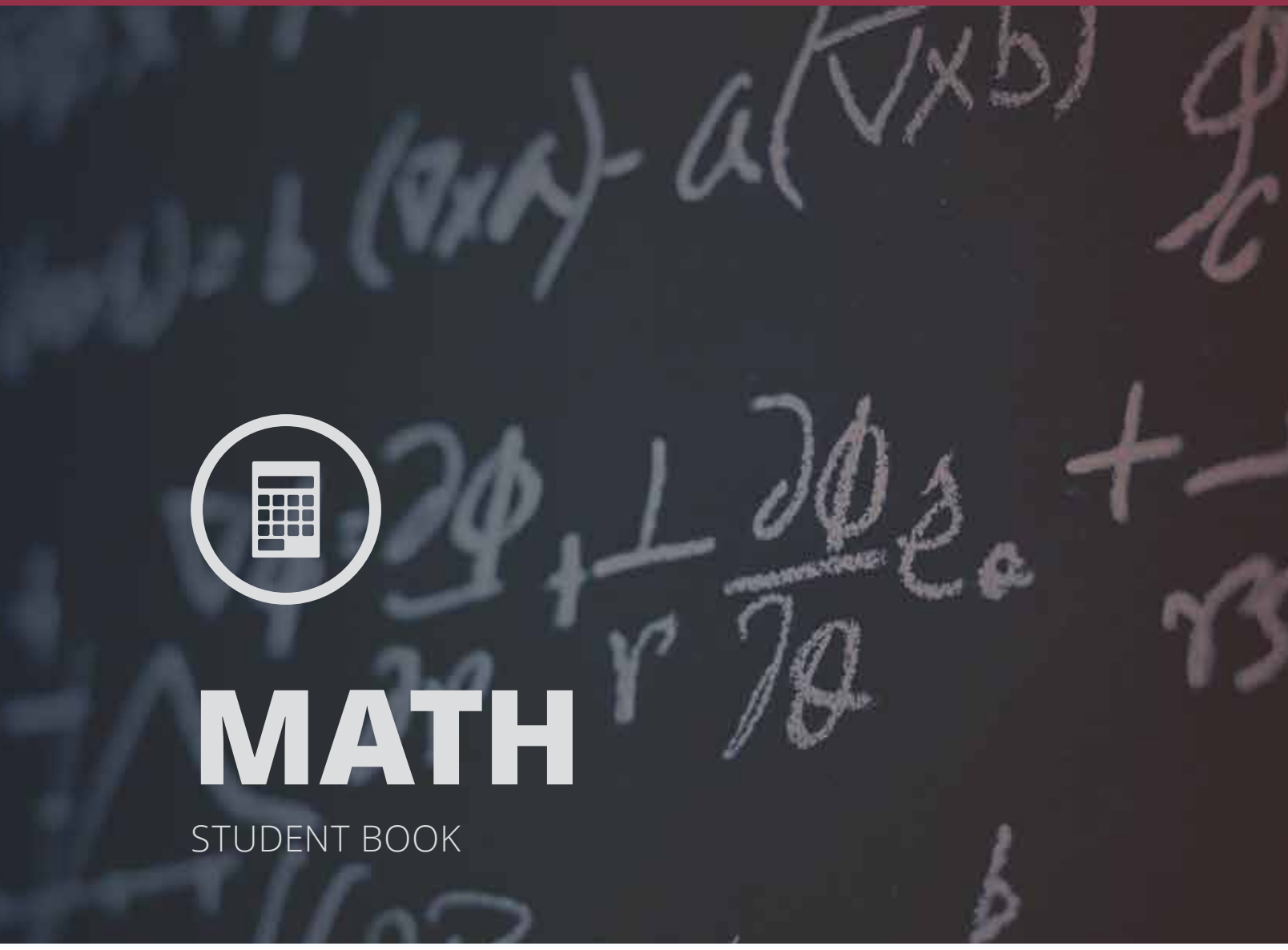

MATH

STUDENT BOOK

9th Grade | Unit 1

Alpha Omega
PUBLICATIONS

Math 901

Variables and Numbers

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Variables and Numbers

INTRODUCTION

This LIFEPAC® is your introduction to a system of mathematics unlike the arithmetic you learned in the elementary grades. In arithmetic you were taught the rules that govern the four operations of the system—addition, subtraction, multiplication, and division; and you were told which operation to perform on a given set of numbers to get the answer. Here are typical examples of exercises in arithmetic: 4 + 3 = 7, 7 – 2 = 5, 13 x 4 = 52, 12 ÷ 3 = 4.

Algebra, like geometry, trigonometry, and calculus, is another of several mathematical *systems*. Like arithmetic, it has its own operating rules. Unlike arithmetic, algebra often requires you to find the value of one of the numbers—the *unknown*—in an exercise. Sometimes, you will have to decide for yourself what operation to use, and sometimes several operations will be used. Toward the end of this LIFEPAC, you will learn how to apply the arithmetic operations to numbers less than zero—the negative numbers.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Identify bases, exponents, constants, variables, numerical coefficients, terms, sums, and products.
2. Simplify algebraic expressions when possible.
3. Evaluate algebraic expressions.
4. Translate algebraic expressions.
5. Perform operations with signed numbers.

1. EXPRESSIONS

The expression 8 + 3 is a *numerical expression*: *numerical* because it consists of numbers; *expression* because it expresses an operation, in this case addition.

In algebra letters of the alphabet are used to represent numbers. These letters are referred to either as *unknowns* or as *variables*. An expression that contains a variable, such as $n + 3$, is an *algebraic expression*. Learning to handle algebraic expressions is the first step in this new system of mathematics. You will have an opportunity in this section to review and practice basic number skills and then to apply those skills in simplifying expressions by the distributive property.

OBJECTIVES

Review these objectives. When you have completed this section, you should be able to:

1. Identify bases, exponents, constants, variables, numerical coefficients, terms, sums, and products.
2. Simplify algebraic expressions when possible.
3. Evaluate algebraic expressions.
4. Translate algebraic expressions.

VARIABLES

If expressions, whether numerical or algebraic, imply addition, they are called *sums*; if they imply subtraction, they are called *differences*; if multiplication, *products*; and if division, *quotients*. These four operations will now be used in evaluating expressions.

SUMS AND DIFFERENCES

In the expression $n + 3$, n and 3 are *addends*. Since we have inserted the plus sign between the letter n and the number 3, the expression is called an *indicated sum*. Its value cannot be determined until we know the value of n.

The expression $n - 3$ means that 3 is to be subtracted from n. Likewise, $3 - n$ means that n is to be subtracted from 3. $n - 3$ is called an *indicated difference*. The expressions $n - 3$ and $3 - n$ are not necessarily equal, because subtraction is an *ordered* operation. We see that $8 - 3$ cannot be $3 - 8$. The *differences* are different.

In an algebraic expression, the letter that represents a number is called a *variable*. In the expression $n + 8$, n is the variable and 8 is the *constant*.

Here are some other models of sums and differences.

$$5 - y,\ x + 6,\ A + 10,\ A + B,\ x + y,\ x - y$$

Simplify. Work from left to right and perform any operation in parentheses first.

Model: 9 + 12 – 3 = 21 – 3 = 18
(2 + 5) – 4 = 7 – 4 = 3

1.1	9 + 6	_____	**1.2**	8 + 13	_____
1.3	5 + 22	_____	**1.4**	17 + 16	_____
1.5	32 + 43	_____	**1.6**	9 + 5 + 4	_____
1.7	3 + 8 + 4	_____	**1.8**	10 + 15 + 4	_____
1.9	17 + 18 + 5	_____	**1.10**	14 + 13 + 7	_____
1.11	10 – 6 + 8	_____	**1.12**	15 – 4 + 1	_____
1.13	17 – 3 – 4	_____	**1.14**	13 – 8 + 10	_____
1.15	28 + 4 – 10	_____	**1.16**	5 + (6 – 4)	_____
1.17	10 + (3 – 2)	_____	**1.18**	29 – (7 + 2)	_____
1.19	(13 + 2) – 8	_____	**1.20**	(50 + 5) – 11	_____

Write the meaning of each of the following expressions.

Model: $x + 10$ The sum of some number x and 10.

1.21 $n + 5$ _____

1.22 $n - 5$ _____

1.23 $x + 8$ _____

1.24 $x - 8$ _____

1.25 $8 - x$ _____

1.26 $5 - y$ _____

1.27 $x + (5 + 7)$ _____

1.28 $x - (8 + 2)$ _____

1.29 $x + (8 - 2)$ _____

1.30 $x + x$ _____

Identify the variable and constant in each of the following expressions and tell if it is a sum, a difference, or neither.

		Variable	Constant	Operation
Model:	$x - 8$	a. *x*	b. 8	c. difference
1.31	$6 + y$	a._____	b._____	c._____
1.32	$N - 8$	a._____	b._____	c._____
1.33	A	a._____	b._____	c._____
1.34	$B - 3$	a._____	b._____	c._____
1.35	$C + 10 + 12$	a._____	b._____	c._____

Write an algebraic expression of each of the following statements.

1.36 The sum of *n* and 6. _____

1.37 The difference of 8 and *n.* _____

1.38 The difference of *n* and 10. _____

1.39 The sum of *n* and itself. _____

1.40 The sum of *n* and the sum of 8 and 6. _____

Sums like 8 + 3 may be written as 3 + 8.

The sum 11 is the same in either case. The ability to interchange addends is called the *commutative property* of addition.

Also, sums like 4 + 2 + 7 may be obtained from (4 + 2) + 7 or from 4 + (2 + 7).

The sum 13 is the same in either case. The ability to change the grouping of the addends is called the *associative property* of addition. These two properties can be used to simplify expressions.

Model: Simplify $3 + x + 7$.

$$\begin{aligned} 3 + x + 7 &= x + 3 + 7 \\ &= x + (3 + 7) \\ &= x + 10 \end{aligned}$$

Simplify.

1.41 $x + 7 + 8$

_____

1.42 $7 + x + 3$

_____

1.43 $9 + 7 + n$

_____

1.44 $x + 15 - 4$

_____

1.45 $(20 + 2) + r$

_____

1.46 $8 + r - 4$

_____

1.47 $15 + x + 10 - 4$

_____

1.48 $(15 - 10) + n$

_____

1.49 $5 + n + (15 - 2)$

_____

1.50 $1.5 + 3.82$

_____

1.51 $17.25 + 3.9$

_____

1.52 $19.62 + 8.33 + 5.7$

_____

1.53 $1.005 + 3.54$

_____

1.54 $73.05 + 8.006$

_____

1.55 $15.63 + 7.956 + 82.735$

_____

1.56 $25.63 - 8.23$

_____

1.57 73.543 – 23.683

_____

1.58 28.543 – 14.26 – 3.65

_____

1.59 x + 6.2 + 8.5

_____

1.60 7.5 + n + 9.63

_____

1.61 81.56 + n – 2.55

_____

1.62 7.95 – 3.86 + N

_____

1.63 22.6 + x – 11.3 + 1.2

_____

1.64 77.65 – 15.56 + x + 1.2

_____

PRODUCTS

The numerical expression 7 + 7 can be renamed several ways, one of which is 2 times 7. We wish to omit the (x) as a times sign. In algebra we will use the dot, 2 • 7, or the parentheses, (2)(7). Therefore, the product of 6 and 9 will be written as 6 • 9 or (6)(9). Likewise, if one of the *factors is literal* — the n in 7 times n — we will write the product as 7n. The dot or parentheses are not to be used when writing literal products.

- **Models:** 6 • 4, 6x, 5n, 15r, r17, A15

Product expressions such as r • 17 and A • 15 are to be written with the constant preceding the variable, 17r and 15A. The constant preceding the variable in a product is called a *numerical coefficient*.

Find the product of each of the following expressions.

1.65	6 • 5	_____	**1.66**	8 • 4	_____
1.67	7 • 6	_____	**1.68**	9 • 8	_____
1.69	4 • 10	_____	**1.70**	(5)(10)	_____
1.71	(15)(12)	_____	**1.72**	(3)(50)	_____
1.73	(40)(5)	_____	**1.74**	(70)(20)	_____
1.75	6(15)	_____	**1.76**	7(22)	_____
1.77	8(15)	_____	**1.78**	16(30)	_____
1.79	10(23)	_____	**1.80**	5 • 7 • 8	_____
1.81	12 • 5 • 8	_____	**1.82**	3(4)(5)	_____
1.83	6(2)(5)	_____	**1.84**	15(2)(8)	_____

Name the numerical coefficient of each of the following expressions.

1.85	$6x$	_____	**1.86**	$5n$	_____
1.87	$22r$	_____	**1.88**	$16p$	_____
1.89	$13q$	_____	**1.90**	$8 \cdot 2N$	_____
1.91	$3 \cdot 2x$	_____	**1.92**	$7.2r$	_____
1.93	$9(14)P$	_____	**1.94**	$2(3)(6)q$	_____

In the operation 5 • 7, the product is the same if the expression is changed to 7 • 5. That is, 5 • 7 = 7 • 5. The ability to interchange factors is called the *commutative property* of multiplication. Also, the *associative property* of multiplication allows you to change the grouping of the factors.

These two properties can be used to simplify expressions. When more than one variable is used, the letters are to be written in alphabetical order.

Model 1: Rewrite $B \cdot 5 \cdot A$

$$\begin{aligned} B \cdot 5 \cdot A &= 5 \cdot B \cdot A \\ &= 5 \cdot (B \cdot A) \\ &= 5 \cdot (A \cdot B) \\ &= 5AB \end{aligned}$$

Model 2: Rewrite $7 \cdot K \cdot 5 \cdot H$

$7 \cdot K \cdot 5 \cdot H$ may be rewritten as $(7 \cdot 5)(H \cdot K)$ using the commutative and associative properties; thus, the simplified form is $35HK$.

Simplify. Remember: When more than one variable is used, the letters are to be written in alphabetical order. Also, no dots are to be shown in the final answers.

1.95 $6 \cdot x \cdot 7$ _____

1.96 $5 \cdot P \cdot 2$ _____

1.97 $3 \cdot S \cdot R$ _____

1.98 $8 \cdot x \cdot 2 \cdot y$ _____

1.99 $a \cdot c \cdot 2 \cdot 5$ _____

1.100 $C \cdot 5 \cdot 2 \cdot A$ _____

1.101 $4 \cdot Q \cdot 2 \cdot P$ _____

1.102 $10 \cdot K \cdot 2$ _____

Write the meaning of each of the following expressions.

Model: $4A$ The product of 4 and some number.

Model: $10N - 2$ The difference between ten times some number and 2.

1.103 $7n$ _____

1.104 $6P$ _____

1.105 $8N + 5$ _____

1.106 $7 + 2x$ _____

1.107 $12x - 10$ _____

1.108 $52 - 25x$ _____

EXPONENTS

The numerical expression 5 times 5 may be written as 5^2. The 2 is called an exponent. The exponent is a counter for the number of repeated factors.
Thus $6 \cdot 6 = 6^2$ and $8 \cdot 8 \cdot 8 = 8^3$.

In the case of literal expressions, we have $x \cdot x = x^2$ and $A \cdot A \cdot A = A^3$. Conversely, x^3 means $x \cdot x \cdot x$, or three factors of x.

Models:

$x^2 = x \cdot x$	$P^2 = P \cdot P$	$(ab)^2 = ab \cdot ab$
$x^3 = x \cdot x \cdot x$	$P^3 = P \cdot P \cdot P$	$(ab)^3 = ab \cdot ab \cdot ab$
$x^4 = x \cdot x \cdot x \cdot x$	$P^4 = P \cdot P \cdot P \cdot P$	$(ab)^4 = ab \cdot ab \cdot ab \cdot ab$

x^2 is read, "The square of x" or "x squared."
x^3 is read, "The cube of x" or "x cubed."
x^4 is read, "The fourth power of x" or "x to the fourth."

x^n is an *indicated power*. x is called the base, and n is the *exponent* of the base.

In each case, identify the base and the exponent of the indicated power.

Model: 3^8 base = 3 exponent = 8

		Base	Exponent			Base	Exponent
1.109	2^6			**1.110**	3^9		
1.111	5^{10}			**1.112**	8^3		
1.113	x^6			**1.114**	y^5		
1.115	7^n			**1.116**	9^P		
1.117	15^{x+1}			**1.118**	10^{3x-1}		

Write each of the following expressions in product form.

Model: $A^3 = A \cdot A \cdot A$

1.119	6^3	=		**1.120**	7^4	=	
1.121	x^2	=		**1.122**	y^5	=	
1.123	3^3	=		**1.124**	1^4	=	
1.125	2^5	=		**1.125**	$(\frac{1}{2})^3$	=	
1.127	$(2.5)^2$	=		**1.128**	$(.01)^4$	=	

Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific areas where restudy is needed for mastery.

SELF TEST 1

Multiply (each answer, 3 points).

1.01 $11(2x + 3)$

_____

1.02 $12(5x - 4)$

_____

1.03 $8(3 - 2x)$

_____

Simplify (each answer, 3 points).

1.04 $3x + 3 + 2$

_____

1.05 $5x + 4x + 1$

_____

1.06 $6(x + 2) + 7$

_____

Identify the numerical coefficient of the variable and the constant term (each answer, 2 points).

		Numerical Coefficient	Constant Term
1.07	$6x + 5$	a._____	b._____
1.08	$7x^3 + 2$	a._____	b._____

Identify the base and the exponent for the indicated power (each answer, 2 points).

		Base	Exponents
1.09	5^2	a. _____	b. _____
1.010	8^3	a. _____	b. _____
1.011	$(x^2 + 3)$	a. _____	b. _____

Evaluate (each answer, 3 points).

1.012 5^3 _____

1.013 $(3 + 4)^2$ _____

1.014 x^3 for $x = 2$ _____

1.015 $A^2 + B^2$ for $A = 2$, $B = 3$ _____

1.016 $N^2 + 2N + 1$ for $N = 5$ _____

Write in algebraic form (each answer, 5 points).

1.017 8 times the square of a number _____

1.018 The difference of 3 squared and 5 times a number _____

Write in words (each answer, 5 points).

1.019 $x^3 + 4$ _____

1.020 $5x - 2$ _____

Write the required quantities (each answer, 5 points).

1.021 Find 15% of 63 _____

1.022 Find $\frac{5}{8} + \frac{7}{12}$ _____

1.023 Find (0.56)(2.36) _____

1.024 Find $2\frac{3}{8} \cdot 5\frac{3}{4}$ _____

75/93 **SCORE** _____ **TEACHER** _____ _____
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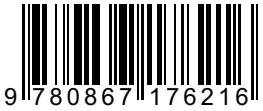

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