



# MATH

STUDENT BOOK

▶ **8th Grade | Unit 6**

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# Math 806

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# Measurement

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## Introduction

This unit covers basic geometric concepts associated with angles, parallel and perpendicular lines, and circles. The properties of various polygons and their angle measures are used to solve for missing angle measures. The special relationship that exists between the sides and angles of right triangles, including a proof of the Pythagorean theorem, is also discussed.

## Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAAC. When you have finished this LIFEPAAC, you should be able to:

- Classify and measure angles and lines.
- Identify and find measures of angles created by transversals.
- Identify parts of circles and their measures.
- Classify polygons and find measures of their interior and exterior angles.
- Classify triangles and use the triangle inequality theorem.
- Classify quadrilaterals and the relationships among them.
- Find side lengths of right triangles using the Pythagorean theorem.

# 1. Angle Measures and Circles

## CLASSIFY AND MEASURE ANGLES

Math is part of our everyday world in everything from balancing checkbooks, to shopping, to measuring a room for new carpet. Math is also a key part of many of the games we play. For instance, the game of pool is nothing but a game of angles. Knowing the correct angle needed to hit the ball is essential to winning a game of pool. One of the best skills you can have if you are a pool player is being able to identify the angle and angle measure needed to sink a shot.

### Objectives

- Identify angles by their measure.
- Classify pairs of angles.
- Find the measure of an angle.



### Vocabulary

**acute angle**—an angle that measures more than  $0^\circ$  but less than  $90^\circ$

**adjacent angles**—two angles that have a common vertex and side but are not overlapping

**complementary angles**—two angles whose sum is  $90^\circ$

**obtuse angle**—an angle that measures more than  $90^\circ$  but less than  $180^\circ$

**right angle**—an angle that measures exactly  $90^\circ$

**straight angle**—an angle that measures exactly  $180^\circ$

**supplementary angles**—two angles whose sum is  $180^\circ$

**vertex**—the point where two line segments, lines, or rays meet to form an angle

**vertical angles**—angles that are opposite from one another at the intersection of two lines; vertical angles are congruent

### Classifying Angles

Angles are a part of our everyday life. Just by looking around the room you are in, you should be able to see hundreds of angles! Each angle or pair of angles has a name and set of properties to define it.

You may remember the terms *acute angle*, *right angle*, *obtuse angle*, and *straight angle*. An acute angle is an angle that measures more than  $0^\circ$  but less than  $90^\circ$ . A right angle measures exactly  $90^\circ$ , while an obtuse angle measures more than  $90^\circ$  but less than  $180^\circ$ . A straight angle, or straight line, measures exactly  $180^\circ$ .

You can easily classify an angle as acute, right, obtuse, or straight just by looking at it. If you remember that a right angle is like a corner on a piece of paper and a straight angle is a straight line, you can tell what type of angle you have without measuring it. If the angle is smaller than the corner of a piece of paper, you have an acute angle. If it is larger than the corner of a piece of paper but smaller than a line, then you have an obtuse angle.

Let's look at how to classify pairs of angles. *Adjacent angles* are two angles that share a common vertex, point, and side. They are next to one another without overlapping. Two types of angles that are often adjacent are *complementary angles* and *supplementary angles*.

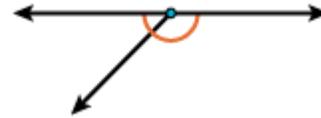
Complementary angles are two angles whose sum is  $90^\circ$ . The two angles can either be adjacent angles or separate angles. The following graphic shows two adjacent, complementary angles.

### Complementary angles:

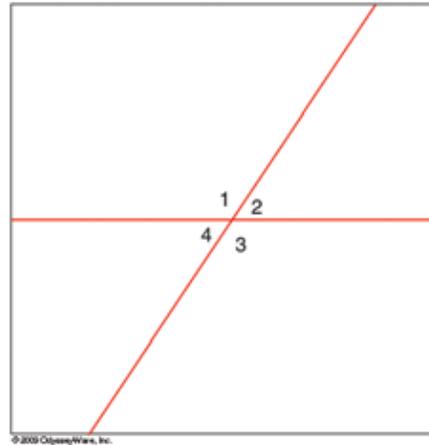


Supplementary angles are two angles whose sum is  $180^\circ$ ; the angles can either be separate or adjacent. The following graphic shows two adjacent, supplementary angles.

### Supplementary angles:



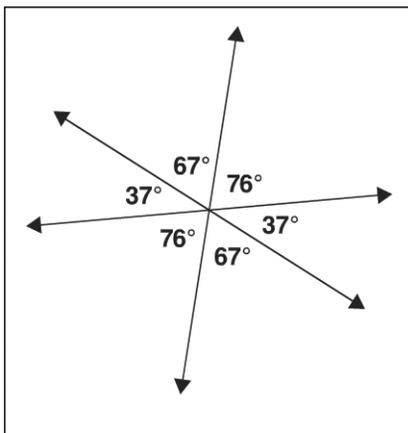
Angles are also created when lines intersect.



When the two lines intersected, they created four angles. The four angles all share a common vertex, where the lines intersect. Some of the angles can also be described as being adjacent. Angle 1 is adjacent to both angle 4 and angle 2. Angle 2 is adjacent to angle 1 and angle 3. Angle 3 is adjacent to angles 2 and 4. Finally, angle 4 is adjacent to angles 3 and 1.

Within these four angles, there are 2 sets of vertical angles. Vertical angles are angles that are opposite one another but share a vertex. Angles 1 and 3 are vertical angles, because they have the same vertex and are opposite one another. The other set of vertical angles are angles 2 and 4. Again, they have the same vertex and are opposite one another.

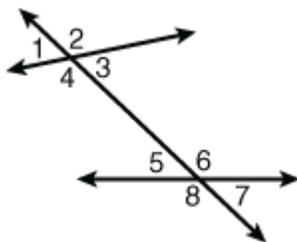
Vertical angles are easily identifiable, because they are found where lines intersect. Look at the following picture. Three lines intersect at one point. The newly created angles are labeled for you.



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Let's look at more sets of vertical angles. See if you can identify them before reading the answers.

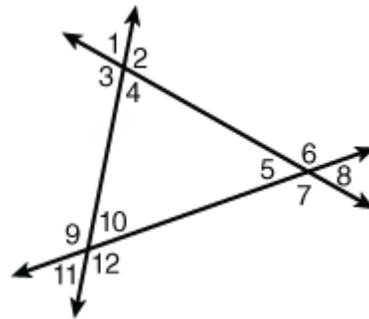
**Example:**



**Solution:**

- ▶ There are four sets of vertical angles:
  - $\angle 1$  and  $\angle 3$
  - $\angle 2$  and  $\angle 4$
  - $\angle 5$  and  $\angle 7$
  - $\angle 6$  and  $\angle 8$

**Example:**



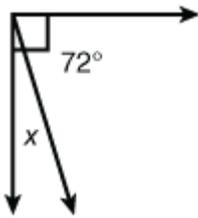
**Solution:**

- ▶ There are 6 sets of vertical angles:
  - $\angle 1$  and  $\angle 4$
  - $\angle 2$  and  $\angle 3$
  - $\angle 5$  and  $\angle 8$
  - $\angle 6$  and  $\angle 7$
  - $\angle 9$  and  $\angle 12$
  - $\angle 10$  and  $\angle 11$

### Measurements of Angles

Now that you know how to classify angles and pairs of angles, we can look into how to find the exact measure of an angle. Remember that complementary angles have a sum of  $90^\circ$ , and supplementary angles have a measure of  $180^\circ$ . Look at the following examples to see how to find the measure of the angle.

**Example:**



**Solution:**

- ▶ The first thing you need to determine is if the pair of angles are complementary or supplementary. The angles are complementary. We can tell this because there is a right angle symbol.
- ▶ We now know that the two angles have a sum of  $90^\circ$ . We also know that one of the angles measures  $x$ , while the other angle measures  $72^\circ$ . We have enough information to set up an equation to find the value of  $x$ .

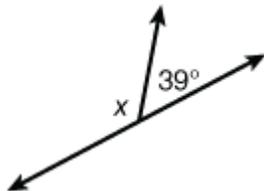
$$x + 72^\circ = 90^\circ \quad \text{Complementary angles sum to } 90^\circ.$$

$$x + 72^\circ - 72^\circ = 90^\circ - 72^\circ \quad \text{Subtract 72 from both sides.}$$

$$x = 18^\circ \quad \text{Complete the subtractions.}$$

- ▶ The second angle in the complementary pair of angles measures  $18^\circ$ .

**Example:**



**Solution:**

- ▶ This time we have a pair of supplementary angles, so we need to set the equation equal to  $180^\circ$ .

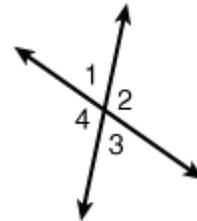
$$x + 39^\circ = 180^\circ \quad \text{Supplementary angles sum to } 180^\circ.$$

$$x + 39^\circ - 39^\circ = 180^\circ - 39^\circ \quad \text{Subtract } 39^\circ \text{ from both sides.}$$

$$x = 141^\circ \quad \text{Complete the subtractions.}$$

Let's move on to vertical angles. You know that vertical angles are angles that are opposite one another and share a vertex. Vertical angles are also congruent, or equal to one another. This means once you know the measure of one, you automatically know the measure of the other.

**Example:**



- ▶  $\angle 1 = 82^\circ$
- ▶ What are the measures of the other angles?

**Solution:**

- ▶  $\angle 1$  is a vertical angle with  $\angle 3$ . Therefore, we now know that  $\angle 3$  equals  $82^\circ$ .
- ▶ We can easily find angles 2 and 4. We know that they are also a set of vertical angles. This means that all we have to do is find the measure of one angle, and we'll know the measure of the other.
- ▶ If we look closely at the image, we see that angle 1 and angle 2 are supplementary, because they form a straight angle. Since supplementary angles equal  $180^\circ$ , angle 1 equals  $82^\circ$ , and angle 2 equals  $x$ , you can now set up an equation.

$$x + 82^\circ = 180^\circ$$

Supplementary angles sum to  $180^\circ$ .

$$x + 82^\circ - 82^\circ = 180^\circ - 82^\circ$$

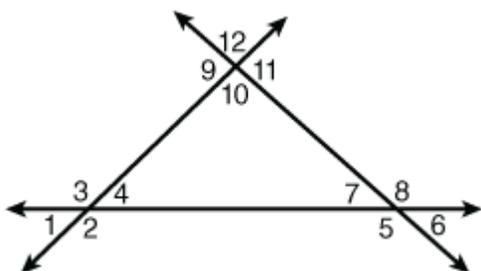
Subtract  $82^\circ$  from both sides.

$$x = 98^\circ$$

Complete the subtractions.

- ▶ Angle 2 measures  $98^\circ$ . That means angle 4 also equals  $98^\circ$ .

### Example:



- ▶  $\angle 2 = 135^\circ$
- ▶  $\angle 8 = 160^\circ$
- ▶  $\angle 11 = 65^\circ$
- ▶ What are the measures of the other angles?

### Solution:

- ▶ The easiest way to solve this problem is to use one point of intersection at a time.
- ▶ Let's begin with the intersection that creates angles 1, 2, 3, and 4. So far, we know that  $\angle 2 = 135^\circ$ . We also know that angles 2 and 3 are vertical angles, which means that they are equal. Finally, we can see that angles 1 and 2 are supplementary. That means they have a sum of  $180^\circ$ .

$$x + 135^\circ = 180^\circ$$

Supplementary angles sum to  $180^\circ$ .

$$x + 135^\circ - 135^\circ = 180^\circ - 135^\circ$$

Subtract  $135^\circ$  from both sides.

$$x = 45^\circ$$

Complete the subtractions.

- ▶ We now know that angle 1 equals  $45^\circ$ . Since angle 1 and angle 4 are vertical angles, we can determine that angle 4 is also equal to  $45^\circ$ .
- ▶ Let's move to the intersection that creates angles 5, 6, 7, and 8. You were told that  $\angle 8 = 160^\circ$ . That means angle 5 also equals  $160^\circ$ , because 8 and 5 are vertical angles. You can see that angles 5 and 6 are supplementary angles, so you know their sum is equal to  $180^\circ$ .

$$x + 160^\circ = 180^\circ$$

Supplementary angles sum to  $180^\circ$ .

$$x + 160^\circ - 160^\circ = 180^\circ - 160^\circ$$

Subtract  $160^\circ$  from both sides.

$$x = 20^\circ$$

Complete the subtractions.

- ▶ If angle 6 equals  $20^\circ$ , then angle 7 also equals  $20^\circ$ .
- ▶ Finally, let's look at the intersection that creates angles 9, 10, 11, and 12. You know that angle 11 equals  $65^\circ$ , so that means angle 9 also equals  $65^\circ$ . Angles 9 and 10 are supplementary to one another.

$$x + 65^\circ = 180^\circ$$

Supplementary angles sum to  $180^\circ$ .

$$x + 65^\circ - 65^\circ = 180^\circ - 65^\circ$$

Subtract  $65^\circ$  from both sides.

$$x = 115^\circ$$

Complete the subtractions.

- ▶ Angle 10 measures  $115^\circ$ , as well as angle 12.

**Let's Review**

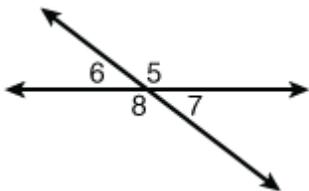
Before going on to the practice problems, make sure you understand the main points of this lesson.

- Identify the types of angles.
  - Classify pairs of angles.
  - Find the measure of angles.
- 

**Complete the following activities.**

- 1.1** If angle 1 measures  $98^\circ$  and angle 2 measures  $72^\circ$ , then the two angles are supplementary.
- True  
 False
- 1.2** Complementary angles have a sum of  $90^\circ$ .
- True  
 False
- 1.3** All pairs of vertical angles are equal.
- True  
 False
- 1.4** Adjacent angles are always supplementary.
- True  
 False
- 1.5** If angle 1 and angle 5 are vertical angles and angle 1 equals  $55^\circ$ , then angle 5 will equal \_\_\_\_.
- $25^\circ$                         $55^\circ$                         $125^\circ$                        can't be determined
- 1.6** If each of two complementary angles has the same measure, then each angle will equal \_\_\_\_.
- $180^\circ$                         $90^\circ$                         $45^\circ$                         $22.5^\circ$

1.7 In the following diagram, angle 7 equals  $61^\circ$ .



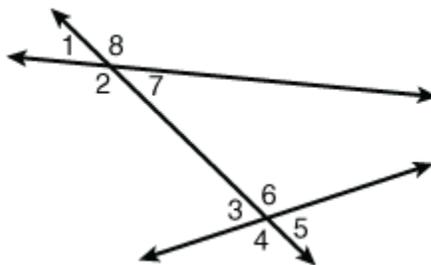
What is the measure of angle 8?

- $29^\circ$   
  $61^\circ$   
  $119^\circ$   
  $129^\circ$

1.8 If two angles are supplementary and one of the angles measures  $38^\circ$ , what is the measure of the larger angle?

- $62^\circ$                         $152^\circ$                         $52^\circ$                         $142^\circ$

Use this illustration for the following questions.



1.9 Angle 3 is equal to angle \_\_\_\_\_.

1.10 Angle 2 is equal to angle \_\_\_\_\_.

1.11 Angle 7 is equal to angle \_\_\_\_\_.

1.12 Angle 6 is equal to angle \_\_\_\_\_.

## PERPENDICULAR AND PARALLEL LINES, PART 1

Pictured is the Hearst Tower in New York. The tower earned the 2006 Emporis Skyscraper Award for the best skyscraper in the world completed that year. It was also the first “green” building in New York.

Take a closer look at the triangular framing design on the outside of the building. Can you identify the different types of angles that are created? Can you also identify the different types of lines that exist?



### Objectives

- Identify lines as parallel, intersecting, or perpendicular.
- Identify a transversal and the angles it creates.
- Find the measure of angles created by a transversal.

### Vocabulary

**alternate exterior angles**—two outside angles that lie on different sides of a transversal

**alternate interior angles**—two inside angles that lie on different sides of a transversal

**corresponding angles**—two angles in the same position on different lines

**exterior angles**—outside angles

**interior angles**—inside angles

**intersecting lines**—lines that share one point

**parallel lines**—lines that never cross one another and are the same distance apart at all times

**perpendicular lines**—lines that intersect and create right angles

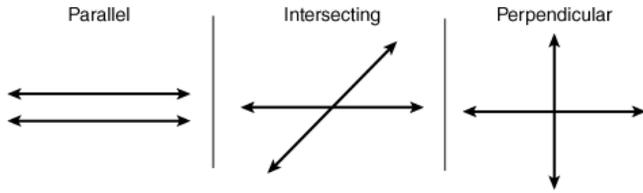
**transversals**—lines that intersect two or more lines to create angles

### Classifying Lines

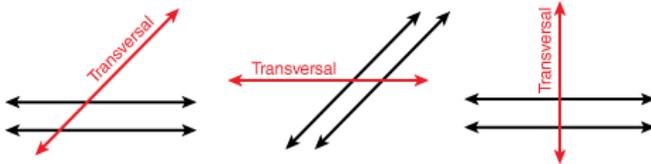
Every set of lines can be classified as parallel, perpendicular, or intersecting. They are often easy to tell apart, just by looking at them.

*Parallel lines* are two lines that never intersect. They are also always the same

distance apart. *Intersecting lines* are lines that cross one another at one point. Finally, *perpendicular lines* are lines that cross one another and create four right angles at the point of intersection.

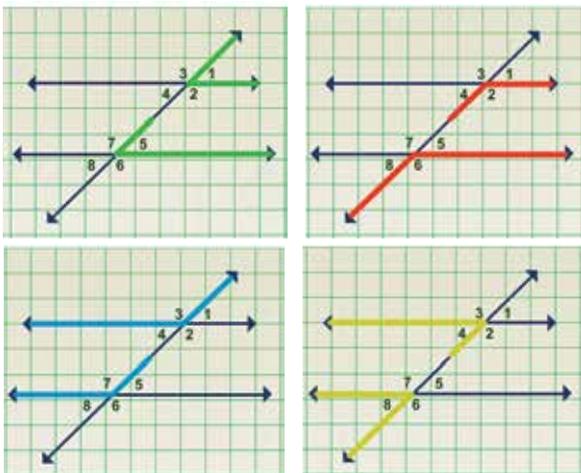


Sometimes, another line will cross a set of lines in a different direction. This individual line is called a *transversal*. A transversal is a line that crosses two or more lines and creates angles. Take a look at some examples of transversals. The transversals are the red lines in each picture. Notice that the transversal can cut horizontally, vertically, or diagonally.

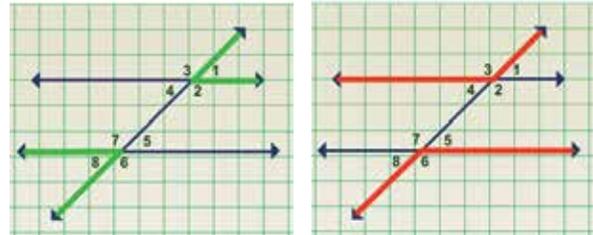


When a transversal cuts across a set of two lines, it creates eight angles, four angles per intersection. The angles created on the two lines have a relationship with one another.

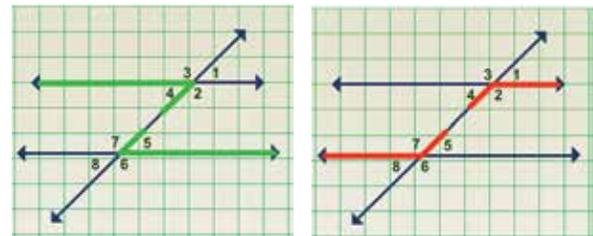
One type of angle that is created is called *corresponding angles*. Corresponding angles are angles that are in the same position but on different lines. There are four sets of corresponding angles.



The transversal also creates two sets of angles known as *alternate exterior angles*. Alternate exterior angles are angles that are on the opposite sides of the transversal, opposite line, and on the outside of the lines.



The last two sets of angles created by the transversal are *alternate interior angles*. Alternate interior angles are angles that are on the opposite sides of the transversal, opposite line, and are on the inside of the lines.



When a transversal cuts across a set of parallel lines, the angles that are created have a special relationship. Corresponding angles, alternate interior angles, and alternate exterior angles are all congruent within their own pairs.

---

**Key point!** The sets of angles only become congruent when a transversal cuts across a set of parallel lines!

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# SELF TEST 1: Angle Measures and Circles

Complete the following activities (6 points, each numbered activity).

1.01 Match each picture with its correct label.

complementary angles

acute angle

straight angle

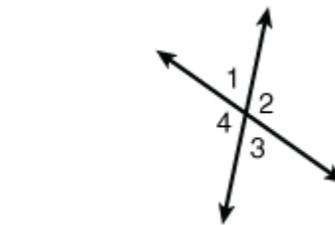
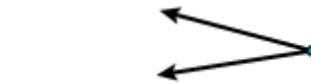
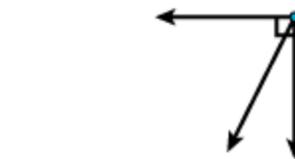
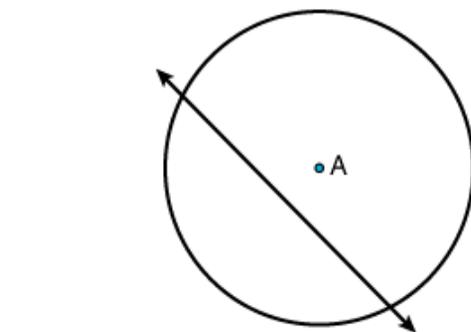
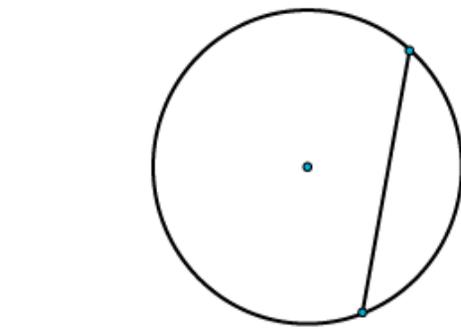
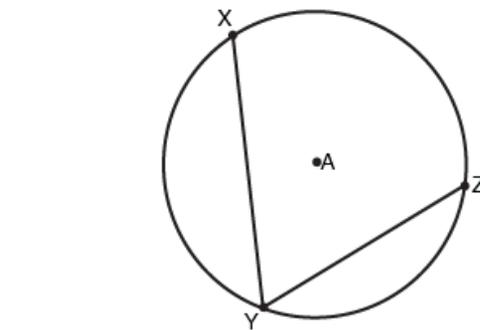
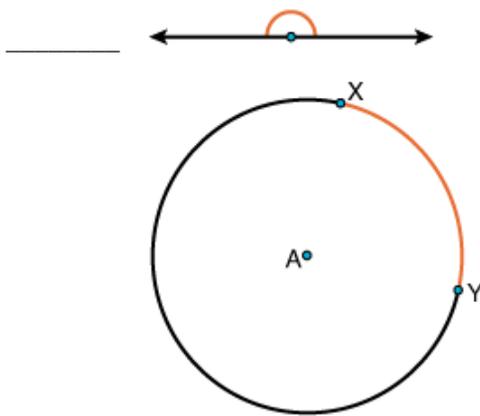
intersecting lines

minor arc

inscribed angle

secant

chord





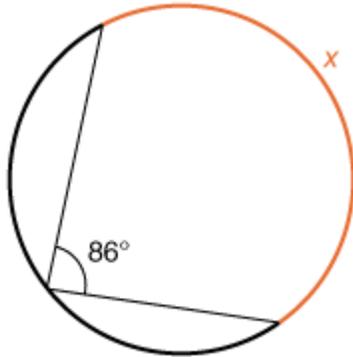
**1.09** If two angles are complementary, then the sum of their angles is equal to \_\_\_\_.

each other

$90^\circ$

$180^\circ$

**1.010** What is the measure of  $x$ ?



$43^\circ$

$86^\circ$

$172^\circ$

$344^\circ$

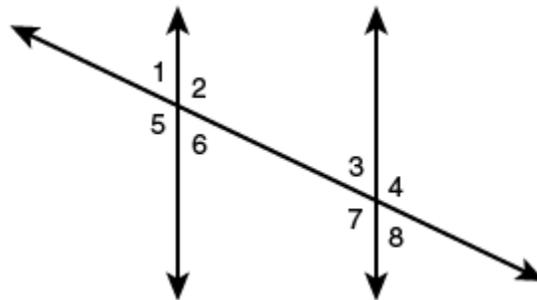
**1.011** If one angle of a set of alternate interior angles on parallel lines measures  $77^\circ$ , then the other angle also equals  $77^\circ$  \_\_\_\_.

always

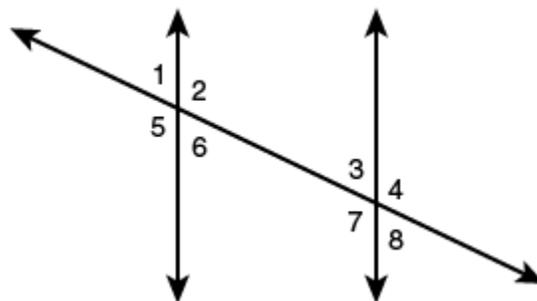
sometimes

never

**1.012** In the graphic, one pair of vertical angles is \_\_\_\_\_ .



**1.013** In the image, the corresponding angle to angle 1 is angle \_\_\_\_\_ .



**1.014** If one angle of a set of supplementary angles measures  $77^\circ$ , then the other angle measures \_\_\_\_.

$13^\circ$

$103^\circ$

$77^\circ$

$283^\circ$

**1.015** If one angle of a set of vertical angles measures  $63^\circ$ , the sum of the vertical angles is \_\_\_\_\_ .

	<b>SCORE</b> _____	<b>TEACHER</b> _____	initials	date
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