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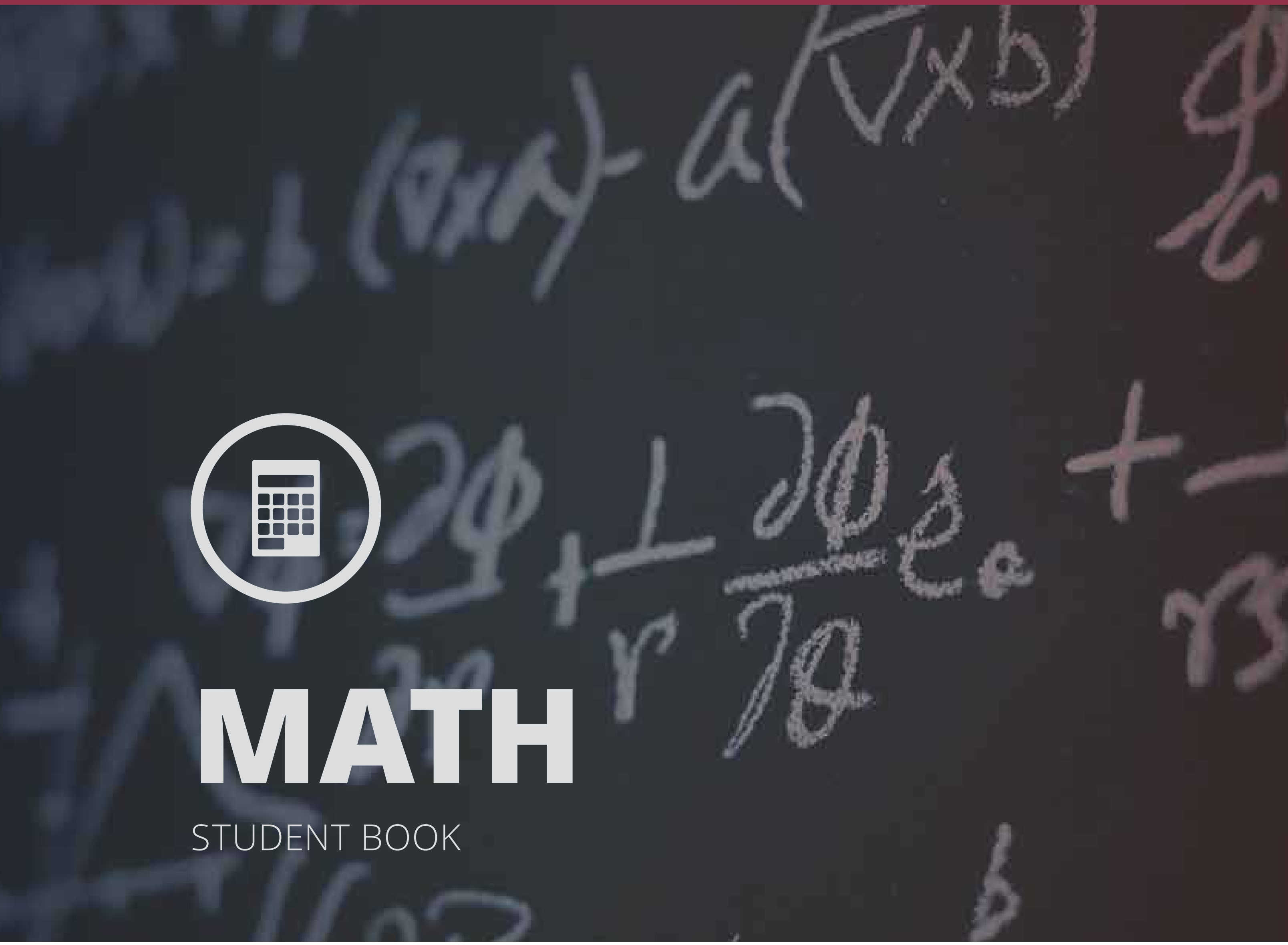

MATH

STUDENT BOOK

8th Grade | Unit 4

Alpha Omega
PUBLICATIONS

Math 804
Proportional Reasoning

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Proportional Reasoning

Introduction

In this unit, ratio and proportion are defined and the different notations for ratio are given. Proportions are used to solve problems, such as unit pricing and rate. Students convert between fractions, decimals, and percents. Students also learn how things are measured indirectly. Word problems require students to use their knowledge of similar figures to set up and solve a proportion. Lastly, students use their knowledge of similar figures and scales to solve problems involving scale drawings.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Use proportions to solve for a missing value.
- Solve direct variation problems.
- Convert and compare fractions, decimals, and percents.
- Solve percent problems.
- Convert customary units.
- Convert metric units.
- Use similar figures to solve for a missing measure and to measure indirectly.

1. Proportions

Proportions

A ratio is used to compare two numbers and a proportion says that two ratios are equal.

Objectives

- Write ratios and proportions.
- Determine if an equation is a proportion.
- Solve for a missing value in a proportion.

Vocabulary

cross multiplication—multiplying diagonally across the equal sign in a proportion
proportion—a statement that says two ratios are equal
ratio—a comparison of two quantities or numbers as a quotient

Ratios

A caterer is calculating the amount of food he needs to purchase for an upcoming party. According to his serving chart, he will need 2 ounces of a certain kind of cheese for every 3 people at the party. If there are supposed to be 102 people attending the party, how many ounces of cheese does the caterer need to purchase?

A *ratio* is a comparison of two quantities or numbers. The caterer needs to compare the amount of cheese to the number of people it will serve. This ratio is 2 to 3, or 2 ounces for every 3 people.

Connections! Do you remember the definition of a *rational* number? It gets its name from the word ratio. Any number that can be expressed as the ratio of two integers is considered rational.

Ratios may be expressed by using the word “to,” a colon, or a division symbol. And, they should always be expressed in lowest terms. Each of the following ratios is equivalent, or means the same thing:

- 2 to 3
- 2:3
- $\frac{2}{3}$
- 2 ÷ 3

It is important to write the ratio in the correct order. If you are asked to find the ratio of ounces of cheese to people, then the number of ounces must be the first number, or in the numerator, and the number of people must be the second number, or in the denominator. $\frac{2}{3}$ is not equivalent to $\frac{3}{2}$. The ratio 3 to 2 would be stating that you need 3 ounces of cheese for every 2 people, causing the caterer to order much more than is needed!

Reminder! The second number, or denominator, in a ratio cannot be zero, because division by zero is undefined.

Although ratios are usually written as a fraction, ratios are *not* the same as fractions. Fractions can be expressed as mixed numbers because fractions represent how many parts of a whole that you have. Ratios represent a comparison of *two* numbers, so they should not be expressed as mixed numbers. A single part of the ratio can be a mixed number, but the entire ratio should *always* be expressed as a proper or improper fraction.

For example! You can express the fraction $\frac{3}{2}$ of pizza as $1\frac{1}{2}$ pizzas, since it is a single number. By contrast, you cannot express the ratio 3 pizzas for every two people ($\frac{3}{2}$) as $1\frac{1}{2}$ because the mixed fraction leaves out the number of people. However, you can convert the $\frac{3}{2}$ ratio of pizzas to people to $\frac{1\frac{1}{2}}{1}$, since 3 pizzas for every 2 people is the same as $1\frac{1}{2}$ pizzas for every 1 person.

Let's look at some other ratios. Suppose you know that a jar has only nickels and dimes in it. And, the ratio of nickels to dimes is 4 to 2. What other ratios could you state from that information? Well, the ratio of dimes to nickels would be the other way around, or 2 to 4. That's an easy one. What about the ratio of nickels to the total number of coins in the jar? You know from the ratio that for every 4 nickels, there are 2 dimes. So, we can say that for every 6 coins (4 + 2) there are 4 nickels and 2 dimes. Now we can form two more ratios! The ratio of nickels to coins is 4 to 6, and the ratio of dimes to coins is 2 to 6. Let's try another one.

Example:

- In a book club, the ratio of women to total members is 4 to 7. What is the ratio of women to men?

Solution:

- The given ratio tells us that for every 7 members, there are 4 women. That means that the remaining 3 members (7 - 4) must be men. So, the ratio of women to men is 4 to 3.

So, how does being able to write a ratio help us to solve the caterer's problem from the beginning of the lesson?

Since we know the ratio of ounces to people has to be 2 to 3, we can write an equivalent ratio that will help us find the number of ounces for 102 people. Again, it is very important that the order of the ratios be consistent. Both ratios should express the number of ounces to the number of people.

$$\frac{\text{ounces of cheese}}{\text{people}} = \frac{2}{3} = \frac{n}{102}$$

The letter *n* is used to represent the unknown value that we are trying to find. So, now we can solve for *n* to find the number of ounces of cheese the caterer needs. What would we multiply the 3 by in order to get 102? 102 divided by 3 is 34, so we can rename this fraction by multiplying both the numerator and the denominator by 34.

$$\frac{2}{3} \cdot \frac{34}{34} = \frac{68}{102}$$

The caterer will need 68 ounces of cheese.

Proportions

We just used a *proportion* to solve the caterer's problem. A proportion is a statement where two ratios are equal. The equation $\frac{2}{3} = \frac{68}{102}$ is a proportion.

Some other examples of proportions are:

- $\frac{1}{2} = \frac{2}{4}$
- $\frac{2}{3} = \frac{6}{9}$
- $\frac{3}{5} = \frac{6}{10}$

Look at the following equations. These are *not* proportions. Can you see why?

$\frac{1}{2} = \frac{1}{4}$ • The numerator stayed the same but the denominator was multiplied by 2.

$\frac{2}{3} = \frac{4}{9}$ • The numerator was multiplied by 2, but the denominator by 3.

If two ratios are not equivalent, the equation is not a proportion.

Example:

- ▶ Is $\frac{6}{16} = \frac{9}{24}$ a proportion?

Solution:

- ▶ Both ratios can be reduced to $\frac{3}{8}$.
- ▶ The ratios are equivalent, so it is a proportion.

Think about it! The fraction bar represents division. A different way to determine if two ratios are equivalent is to divide the numerators by the denominators. If the two quotients are equal, then the ratios are equivalent. In this example, 6 ÷ 16 = 0.375, and 9 ÷ 24 = 0.375

Cross Multiplication

Another approach to determining if an equation is a proportion is to use *cross multiplication*. By multiplying diagonally across the equal sign, we can determine if two ratios are equivalent.

In a proportion, the results of cross multiplication are always equal.

Knowing this fact gives us another option for solving the previous example, $\frac{6}{16} = \frac{9}{24}$. Try cross multiplying.

- $\frac{6}{16} = \frac{9}{24}$
- $6 \cdot 24 = 144$ • Cross product of 6 and 24.
- $16 \cdot 9 = 144$ • Cross product of 16 and 9.

The cross products both equal 144, so the equation is a proportion. If the products are not equal when you cross multiply, then the equation is not a proportion.

Example:

- ▶ Determine if $\frac{2}{3} = \frac{4}{5}$ is a proportion.

Solution:

- ▶ Cross multiply.

$\frac{2}{3} = \frac{4}{5}$	Original equation.
$2 \cdot 5 = 10$	Cross product of 2 and 5.
$3 \cdot 4 = 12$	Cross product of 3 and 4.
$10 \neq 12$	The cross products are unequal.

- ▶ The cross products are not equal, so the ratios are not equivalent. This is not a proportion.

Let's see how cross multiplication can be used to solve the problem from the beginning of this lesson.

We had a ratio of cheese to people that was 2 to 3. We needed to find the amount of cheese for 102 people. The proportion we set up was $\frac{2}{3} = \frac{n}{102}$. We know that using cross multiplication, the products have to be equal. So, set up an equation showing this.

$$2 \cdot 102 = 3n \quad \text{Cross multiply.}$$
$$204 = 3n \quad \text{Simplify.}$$
$$68 = n \quad \text{Divide both sides by 3.}$$

When we solve the equation, we find that $n = 68$. Our answer shows that the caterer needs to order 68 ounces of cheese for the upcoming party. If you look back in the lesson, you'll see that this is the same answer we got before. In math, you can often solve problems in more than one way!

Let's look at a couple more examples.

Example:

- Joe's basketball team's win to loss ratio was 5 to 2. If they lost 4 games, how many games did they win? How many total games did they play?

Solution:

- Set up a proportion using w to represent the number of games they won.

$$\frac{5}{2} = \frac{w}{4} \quad \text{Equation for the proportion.}$$
$$20 = 2w \quad \text{Cross multiply.}$$
$$10 = w \quad \text{Solve the equation.}$$

- The team had 10 wins this season. To find the total number of games played, add the number of wins to the number of losses. Ten wins plus four losses equals 14 total games played.

Be Careful! Make sure that the two ratios are set up consistently. The number of wins should be in the numerator of both ratios, and the number of losses should be in the denominator of both ratios.

Example:

- Six is to $1\frac{1}{3}$ as nine is to a.
- Find a.

Solution:

- Six is to $1\frac{1}{3}$ represents the first ratio. Nine is to a represents the second ratio. The word "as" tells us that these two ratios are proportional.

$$\frac{6}{1\frac{1}{3}} = \frac{9}{a} \quad \text{Equation for the proportion.}$$
$$6a = 1\frac{1}{3} \cdot 9 \quad \text{Cross multiply.}$$
$$6a = \frac{4}{3} \cdot \frac{9}{1} \quad \text{Convert the mixed number } (1\tfrac{1}{3}) \text{ to an improper fraction.}$$
$$6a = \frac{36}{3} \quad \text{Multiply the fractions.}$$
$$6a = 12 \quad \text{Simplify the fraction.}$$
$$a = 2 \quad \text{Divide both sides by 6.}$$

This might help! Remember that to multiply mixed numbers, you first need to change each number to an improper fraction. Then, multiply straight across.

In addition to "as," a double semi-colon is also used to show a proportion. Here are some other examples of what proportions can look like.

- a is to b as c is to d
- $a : b$ as $c : d$
- $a : b :: c : d$

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- Ratios show a comparison between quantities.
- Two equivalent ratios form a proportion.
- In a proportion, the products of cross multiplying are always equal.
- The order of the quantities, in both ratios and proportions, is very important.

Complete the following activities.

1.1 Solve for the variable in the following proportion. $\frac{4}{7} = \frac{x}{161}$
x = _________

1.2 Solve for the variable in the following proportion. $\frac{3.6}{y} = \frac{1.2}{2}$
y = _________

1.3 Solve for the variable in the following proportion. $\frac{5}{25} = \frac{m}{125}$
m = _________

1.4 Solve for the variable in the following proportion. $n : \frac{1}{2}$ as 6 : 1
n = _________

1.5 Solve for the variable in the following proportion. $\frac{1}{4}$ is to $1\frac{1}{4}$ as 2 is to b
b = _________

1.6 Two numbers are in the ratio of 2 to 3. If the smaller number is 18, the larger number is _____.
☐ 21 ☐ 27 ☐ 36 ☐ 54

1.7 All of the following are equivalent *except* _____.
☐ 5 is to 3 ☐ $\frac{45}{27}$ ☐ $\frac{15}{5}$ ☐ 7.5:4.5

1.8 In a group of students, the ratio of girls to boys is 3 to 2. If there are 15 girls, how many total students are there?

☐ 10 ☐ 20 ☐ 25 ☐ 30

1.9 On a field trip, there are 12 adults and 14 students. What is the ratio of the number of adults to the total number of people on the field trip?

☐ 6 to 13 ☐ 12 to 14 ☐ 26 to 12 ☐ 6 to 7

1.10 If $2d = 5c$, then all of the following are true *except*.

☐ $\frac{2}{5} = \frac{c}{d}$ ☐ $\frac{5}{2} = \frac{d}{c}$ ☐ $\frac{2}{c} = \frac{5}{d}$ ☐ $\frac{2}{d} = \frac{c}{5}$

1.11 Which of the following is *not* a proportion?

☐ $\frac{1}{4} = \frac{3}{12}$ ☐ $\frac{4}{10} = \frac{12}{20}$ ☐ $\frac{2}{4} = \frac{3}{6}$ ☐ $\frac{8}{6} = \frac{20}{15}$

1.12 If a soccer team won 7 of its 13 games, what was their ratio of wins to losses? Assume that there were no tie games.

☐ 7 to 6 ☐ 7 to 20 ☐ 7 to 13 ☐ 1 to 2

Applications

The customer in the cartoon offered to pay more than the farmer was asking for the apples. Can you see why? In this lesson, you'll learn how to calculate rates, which include unit prices.

Objectives

- Determine unit rate or unit price.
- Use proportional reasoning to solve problems.

Vocabulary

rate—a type of ratio that compares two different kinds of quantities or numbers

unit price—a rate showing the price for 1 item

unit rate—a rate with a denominator of 1; a rate which shows an amount of something compared to 1 of something else

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Rates

This apple farmer would love it if every customer was as naïve as the character in the comic strip. Sure, 85¢ sounds cheap compared to $1.29, but is it really? If both prices were getting us the same number of apples, then 85¢ would definitely be cheaper! But, is 3 apples for 85¢ really a better deal than 5 apples for $1.29? We can use a type of ratio, called a *rate*, to find the answer.

A rate is a ratio that compares two different kinds of numbers or measurements. For example, distance traveled in a certain amount of time is a rate. Or, the cost of a certain weight of something would also be a rate.

One rate that is fairly common is the comparison of miles, or kilometers, to hours. Everyone who has been in a car is probably familiar with how fast the car is going or its rate of speed. Traveling at a rate of 50 miles per hour is faster than a rate of 30 miles per hour. These rates of speed are called *unit rates* because the rate is given for 1 unit of time, an hour.

This might help! The word "per" means to divide. So, "miles per hour" means that miles are in the numerator of the rate and hours are in the denominator. This is the same in unit price, which means "price per item." The price should *always* be in the numerator when calculating unit price.

A *unit rate* is a rate with 1 in the denominator.

In our apple problem, we have two different rates for buying apples: $1.29 for 5 apples and 85¢ for 3 apples. To determine which one is the better rate, we need to express each as a unit rate. When the unit rate is a price, we usually refer to it as a *unit price*. In other words, how much does it cost for 1 apple in each case?

Set up each rate as a ratio in the form of "price per apple." The price should be in the numerator, and the number of apples should be in the denominator. Also, in order to compare the two rates, the prices must be in the same units. So, change 85¢ to $0.85. Once the rates are set up, divide to get the denominator to be 1.

Farmer's Price:	$\frac{\$1.29}{5\text{ apples}}$	**Customer's Offer:**	$\frac{\$0.85}{3\text{ apples}}$
Divide by 5:	$\frac{1.29 \div 5}{5 \div 5}$	**Divide by 3:**	$\frac{0.85 \div 3}{3 \div 3}$
Unit Price:	$\frac{\$0.258}{1\text{ apple}}$	**Unit Price:**	$\frac{\$0.283}{1\text{ apple}}$

The unit prices are $0.258 per apple and $0.283 per apple.

Keep In Mind! Round up when the next place value is greater than or equal to 5. If the next place value is less than 5, keep the number the same. Also, when comparing, make sure to round rates to the same place value and wait until the *end* of the problem to do any rounding; otherwise, the calculation could be incorrect.

So, the farmer was selling the apples for about 26 cents each. The customer offered to pay about 28 cents each. The man would have paid more for the apples at his "bargain price"!

Rates can also be used to find the value of one quantity when another is known. For example, suppose you need to find the cost to buy 100 apples from the farmer at his original price.

Let's use the rate to write a proportion, and then cross multiply to solve.

$$\frac{\$1.29}{5\text{ apples}} = \frac{x}{100\text{ apples}}$$ Set up the proportion.

$$\$1.29(100) = 5x$$ Cross multiply.

$$\$129.00 = 5x$$ Multiply on the left side.

$$\$25.80 = x$$ Divide both sides by 5.

At the rate of $1.29 for 5 apples, it will cost $25.80 for 100 apples.

Let's look at a few more examples that use proportions to solve problems.

Example:

- Sally drinks one and a half cans of soda for every two hours she is at work. At this rate, how many cans of soda does Sally drink in a 40-hour workweek?

Solution:

$\frac{1\frac{1}{2}\text{ cans}}{2\text{ hours}} = \frac{x\text{ cans}}{40\text{ hours}}$	Set up the proportion.
$1\frac{1}{2} \cdot 40 = 2x$	Cross multiply.
$\frac{3}{2} \cdot \frac{40}{1} = 2x$	Change $1\frac{1}{2}$ to an improper fraction.
$60 = 2x$	Complete the multiplication on both sides.
$30 = x$	Divide both sides by 2.

- Sally will drink 30 cans of soda per 40 hours of work.

Be Careful! Notice that on each side of the proportion we have cans to hours. Also, notice that in the previous examples, the proportions have been set up consistently. When you set up a proportion, make sure that the rates are written in the same order.

Example:

- The ratio of boys to girls in a middle school is 4 to 3. If there are 182 students in the school, how many are boys?

Solution:

- The ratio we're given is of boys to girls. Since we're given how many total students there are, we need the ratio of boys to students to set up the proportion we need. There are 4 boys for every 3 girls, so for every 7 students, there are 4 boys. The ratio of boys to students in the school is 4 to 7.

$\frac{4\text{ boys}}{7\text{ students}} = \frac{n\text{ boys}}{182\text{ students}}$	Set up the proportion.
$4 \cdot 182 = 7n$	Cross multiply.
$728 = 7n$	Complete the multiplication on the left side.
$104 = n$	Divide both sides by 7.

- There are 104 boys in the school.

Example:

- During the cross-country season, Megan ran a 3-mile race in $\frac{1}{3}$ of an hour. She ran a 2-mile race in 11 minutes during the track season. In which race did Megan have a faster average speed?

Solution:

- Find Megan's unit rate of speed for each race. Notice that the times were given in two different units of measurement. The units must be the same in order to make an accurate comparison. One option is to change $\frac{1}{3}$ of an hour into minutes.
- There are 60 minutes in an hour, so $\frac{1}{3}$ of 60 or $\frac{1}{3} \cdot 60$ is 20 minutes.
- 3-mile race: $\frac{3\text{ miles}}{20\text{ minutes}} = \frac{3 \div 20}{20 \div 20} = \frac{0.15\text{ miles}}{1\text{ minute}}$
- 2-mile race: $\frac{2\text{ miles}}{11\text{ minutes}} = \frac{2 \div 11}{11 \div 11} = \frac{0.18\text{ miles}}{1\text{ minute}}$
- Megan had a faster average speed in the 2-mile race.

SELF TEST 1: Proportions

Complete the following activities (6 points, each numbered activity).

1.01 If *x*:6 as 3:9, then *x* is equal to __________ .

1.02 *S* varies directly as *T*. If *S* is 20 when *T* is 4, then *T* is __________ when *S* is 30.

1.03 If $\frac{9}{24} = \frac{3}{x}$, then *x* is __________ .

1.04 *Y* is directly related to *X*, and *Y* is 81 when *X* is 27. The constant of variation is __________ .

1.05 The ratio of boys to girls is 5 to 4. There are 80 boys. How many girls are there?

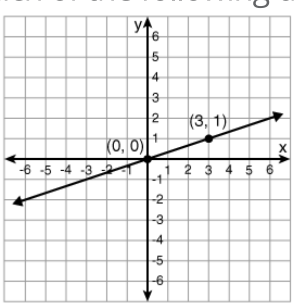
☐ 4 ☐ 16 ☐ 64 ☐ 100

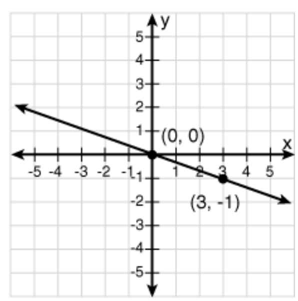
1.06 At what rate is a car traveling, if it goes 157.5 miles in 2.5 hours?

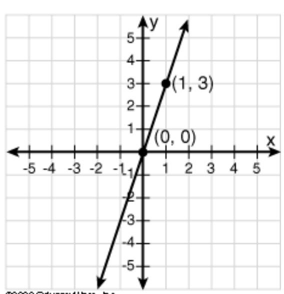
☐ 15 miles per hour ☐ 73 miles per hour

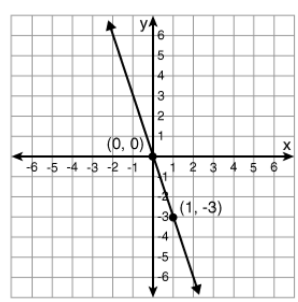
☐ 63 miles per hour ☐ 393.75 miles per hour

1.07 Which of the following direct variations has a constant of variation that is equal to -3?

☐ 

☐ 

☐ 

☐ 

1.08 All of the following ratios are equivalent *except* _____.

☐ 2:3 ☐ $\frac{6}{9}$ ☐ $\frac{6}{4}$ ☐ 8 to 12

1.09 The following table shows a direct variation. Find *y*.

3	15	18	27
y	5	6	9

☐ 0
☐ 1
☐ 2
☐ 3

1.010 Which of the following equations is *not* a proportion?

☐ $\frac{1.5}{3} = \frac{3}{9}$ ☐ $\frac{22}{33} = \frac{4}{6}$ ☐ $\frac{2}{3} = \frac{142}{213}$ ☐ $\frac{14}{7} = \frac{7}{3.5}$

1.011 A certain paint is to be mixed with 3 parts yellow to 2 parts blue. What is the ratio of yellow paint to the total amount of mixed paint?

☐ 3 to 2 ☐ 3 to 5 ☐ 2 to 3 ☐ 3 to 1

1.012 A certain paint is to be mixed with 3 parts yellow to 2 parts blue. Twelve gallons of mixed paint are needed. How many gallons of blue must be used?

☐ 4 ☐ 4.8 ☐ 7.2 ☐ 10

1.013 Which of the following prices is the lowest price per pound?

☐ 2 pounds for $2.75
☐ 4 pounds for $5.60
☐ 3 pounds for $4.05
☐ $1.36 per pound

1.014 Which of the following sets of ordered pairs is *not* a direct variation?

☐ (0, 0); (-2, 4); (3, -6)
☐ (10, 2); (15, 3); (20, 4)
☐ (1, 1); (-2, -2); (3, 3)
☐ (0, 0); (1, 3); (2, 4)

1.015 Which of the following is the equation of a direct variation that has a constant of variation equal to $-\frac{1}{2}$?

☐ $y = x - \frac{1}{2}$ ☐ $-\frac{1}{2}y = x$ ☐ $y = -2x$ ☐ $y = -\frac{1}{2}x$

1.016 A double recipe of cookies calls for 5 cups of flour. Which of the following proportions could be used to find the amount of flour for a triple recipe?

☐ $\frac{5}{2} = \frac{f}{3}$ ☐ $\frac{2}{5} = \frac{f}{3}$ ☐ $\frac{2}{3} = \frac{f}{5}$ ☐ $\frac{3}{2} = \frac{5}{f}$

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