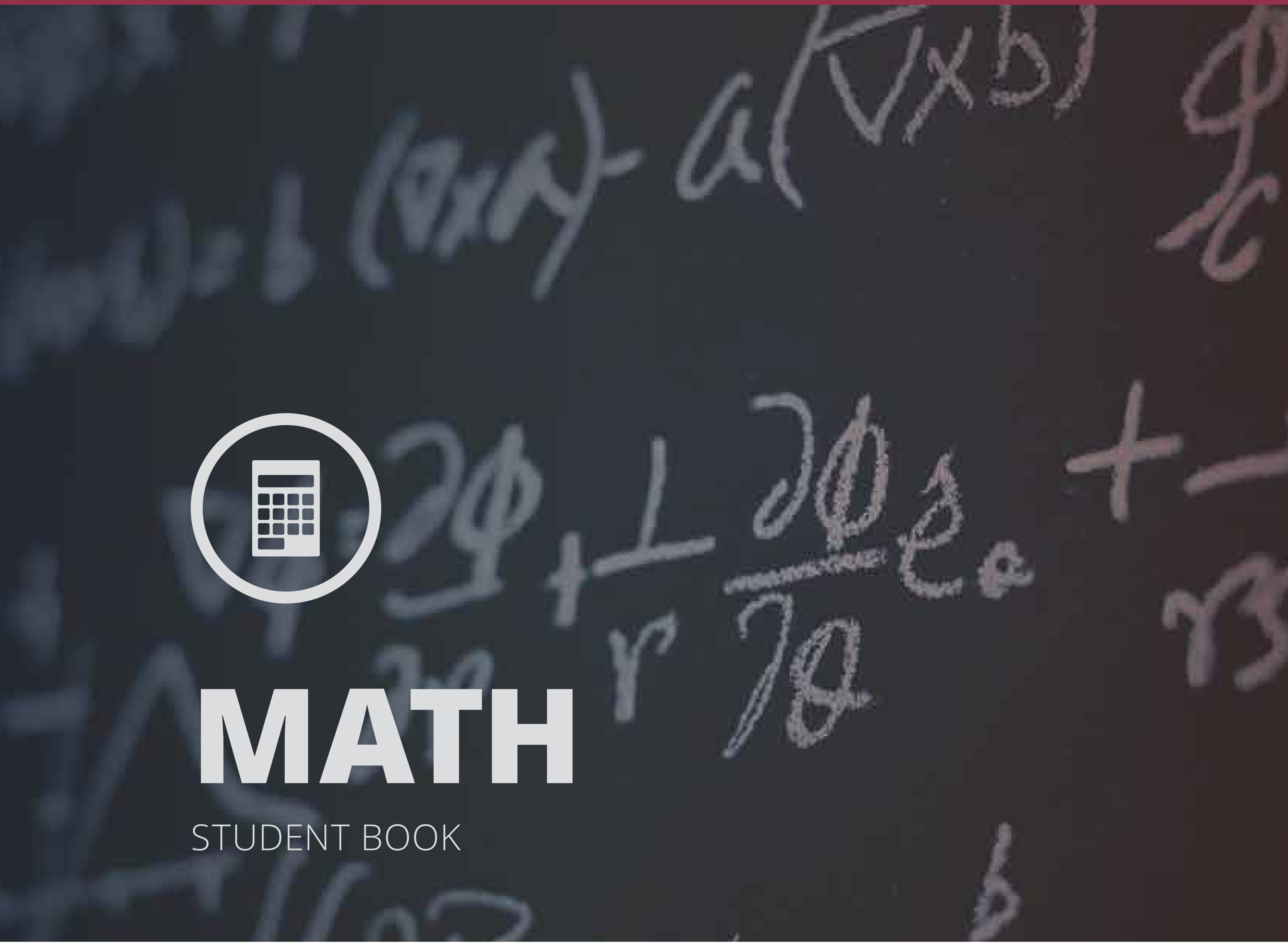
LIFEPAC®



MATH

STUDENT BOOK

8th Grade | Unit 3

Alpha Omega
PUBLICATIONS

Math 803

Modeling Problems with Rational Numbers

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Modeling Problems with Rational Numbers

Introduction

In this unit, students learn to work with rational numbers. They first learn how to find the prime factors of numbers, in preparation for finding greatest common factors, least common multiples, and least common denominators. Students then practice adding, subtracting, multiplying, and dividing with fractions and decimal numbers.

After practicing operations with fractions and decimal numbers, students learn how to solve one-step and two-step equations with rational numbers. Finally, students apply their problem solving skills to finding solutions for inequalities. They also practice graphing inequalities on the number line.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Find the prime factors of numbers and determine the greatest common factors and the least common multiples.
- Perform operations involving positive and negative fractions and decimals.
- Solve one-step and two-step equations with real numbers.
- Solve one-step and two-step inequalities with real numbers.
- Graph inequalities on a number line.

1. Number Theory

Prime Factorization and the GCF

Do you remember what a factor is? A factor is any number that divides evenly into a given number. For example, the factors of 15 are 1, 3, 5, and 15, because each of these numbers divides evenly into 15.

In this lesson, you will learn how to express numbers using their prime factors. You will also use the prime factorizations of two or more terms to find the greatest factor they have in common.

Objectives

- Express the prime factorization of composite numbers and terms in exponential form.
- Determine the greatest common factor using prime factorization.
- Solve problems by applying the greatest common factor.

Vocabulary

composite number—a whole number greater than one that has more than two positive factors

factor—a number that divides evenly into a given number

greatest common factor—the largest number that divides evenly into two or more given numbers

prime factor—a prime number that divides evenly into a given number

prime number—a whole number greater than one that has exactly two positive factors: one and itself

---

Prime Factorization

Let's review how to find the prime factorization of a number.

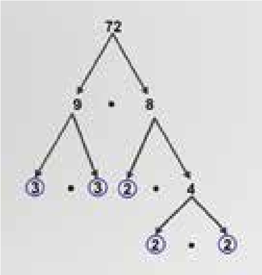
The prime factorization of a number shows what *prime factors* multiply together to get the number. Two methods used to find prime factorization are the *factor* tree and stacked division. We'll look at both methods, and then you can choose the one that you're most comfortable with.

This might help! Remember that a *prime number* is a number greater than one that only has two factors - one and itself. For example, 2, 3, 5, 7, 11, 13, 17, 19, and 23 are all prime numbers. A number greater than one that has more than two factors is called a *composite number*. The number one itself is not a prime or composite number.

Let's find the prime factorization of 72.

Method 1:

Factor Tree

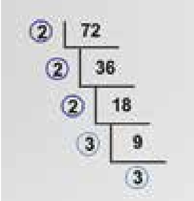


Start with any two factors of 72, and break them down until only prime numbers are left. For example, 8 times 9 is 72. Since 8 and 9 are not prime numbers, we'll need to break them down as well. Let's start with 9. Three times 3 is 9. Three is a prime number so we can stop with each of them. Move to the 8. Two times 4 is 8. Two is a prime number so we can stop with it. But, 4 is composite rather than prime, so we'll need to break it down again. Four is the same as 2 times 2. Two is a prime number so we are finished.

Using the factor tree, the prime factorization of 72 is 2 • 2 • 2 • 3 • 3.

Method 2:

Stacked Division



With stacked division we keep dividing 72 with prime numbers until we are left with a prime number. Start with any easy prime number such as 2, 3, or 5. Seventy-two is divisible by 2 leaving a quotient of 36. Thirty-six is also divisible by 2, leaving us with 18. Eighteen can be divided by 2 leaving us with 9. Nine is not divisible by 2 but it can be divided by 3. The final quotient is 3, a prime number, so we are finished.

The factors of 72 are three 2's and two 3's. or 2^3 times 3^2.

As long as we divide by a prime factor and keep dividing until we have all prime numbers, we will find the prime factorization. Either way we approach the problem, the prime factorization of 72 is 2 • 2 • 2 • 3 • 3, or $2^3 \cdot 3^2$. We can always check our prime factorization by multiplying it out. The product should be the number we started with.

Greatest Common Factors (GCF)

The prime factorization of two or more numbers can help us find their *greatest common factor (GCF)*. The greatest common factor is the largest number that divides evenly into all of the given numbers. There are two methods for finding the GCF.

Keep in mind! The divisibility rules can help you as you look for prime factors. Here's a list to remind you.

2: The last digit is even.
3: The sum of the digits is divisible by 3.
4: The last two digits are divisible by 4.
5: The last digit is 0 or 5.
6: Divisible by both 2 and 3.
9: The sum of the digits is divisible by 9.
10: The last digit is 0.

Suppose we want to find the GCF of 14 and 18. We could list all of the factors of each number, and then look for the greatest factor that they both have. For example, the factors of 14 are 1, 2, 7, and 14. The factors of 18 are 1, 2, 3, 6, 9, and 18. The largest factor that they have in common is 2. So, the GCF of 14 and 18 is 2. This method works, but for larger numbers, it can get very tedious. That is why using the prime factorization method is so helpful. Let's take a look.

First, we'll need to find their prime factorizations.

- $14 = 2 \cdot 7$
- $18 = 2 \cdot 3^2$

Looking at the factorizations, we can see that the only prime factor the two numbers have in common is 2. That means that the greatest common factor, or the GCF, of 14 and 18 is 2.

Sometimes, numbers will have more than one prime factor in common. When this happens, the GCF is the product of all their common prime factors. Check out the following example.

Example:

- Find the GCF of 28, 42, and 70.

Solution:

- First, find their prime factorizations:
  - $28 = 2^2 \cdot 7$
  - $42 = 2 \cdot 3 \cdot 7$
  - $70 = 2 \cdot 5 \cdot 7$

This might help! Even though the 28 has two 2's for prime factors (or 2^2), all three of the numbers would have to have two 2's in common in order for 2^2 to be part of the GCF.

- All three numbers have two factors in common: 2 and 7. To find the GCF, multiply the two common factors together.
  - $2 \cdot 7 = 14$
- The GCF of 28, 42, and 70 is 14.

Sometimes, numbers will have *no* prime factors in common. When this happens, the GCF is 1, and we say that the numbers are relatively prime. For example, the numbers 15 and 8 have no common factors. So, their GCF is 1, and they are relatively prime.

Key point! If two or more numbers have no factors in common, they are relatively prime.

GCF for Variable Expressions

Since variables represent numbers, we can find their GCF in the same way. Look at the following example.

Example:

- Find the GCF of a^2b and ab^2.

Solution:

- First, notice that these expressions are actually prime factorizations.
  - $a^2b = a \cdot a \cdot b$
  - $ab^2 = a \cdot b \cdot b$
- The terms have two common factors, a and b. So, the GCF is $a \cdot b$, or ab.

Key point! Notice that each term was already expressed as a prime factorization. Variable terms are already expressed as a product of their factors in exponential form.

Terms can also have both numbers and variables. If you're asked to find the GCF of such terms, first find the GCF of the numbers. Then, find the GCF of the variables. The complete GCF is the product of the two. Take a look.

Example:

- Find the GCF of $36x^3y^2$ and $24xy^2z$.

Solution:

- Start with the numbers, or constants, and find their prime factorizations.
  - $36 = 2 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot 3^2$
  - $24 = 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3$
- The numbers have two 2's and a 3 in common. So, the GCF of 24 and 36 is $2^2 \cdot 3$, or 12.
- Now, let's look at the prime factors for the variable expressions.
  - $x^3y^2 = x \cdot x \cdot x \cdot y \cdot y$
  - $xy^2z = x \cdot y \cdot y \cdot z$
- The variables have an x and two y's in common. So, the GCF is $x \cdot y^2$, or xy^2.

Be Careful! Remember that the GCF only includes prime factors that are common. With variables, that can be tricky. You are looking for *only* the variables that they both have. Also, for variables that *are* common, look for the smallest exponent because each variable has at least as many as the smallest amount.

- Finally, we multiply the GCFs for the constants, which is 12, and the variable terms, which is xy^2. The GCF of the original terms is $12xy^2$.
- You may be wondering what the GCF is good for. Why do we want to know what the highest common factor is? Fractions! The GCF is very helpful in reducing fractions.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- Any composite number can be written as a prime factorization.
- Variable are already written as a prime factorization.
- We can use the prime factorization of numbers to find the greatest common factor (GCF).
- Terms that have no common factors are relatively prime.

Complete the following activities.

1.1 The prime factorization of 76 is _____.
- ☐ $2 \cdot 38$
- ☐ $2^2 \cdot 19$
- ☐ $2^3 \cdot 7$
- ☐ 76

1.2 The greatest common factor of 114 and 190 is _____.
- ☐ 38
- ☐ 2
- ☐ 19
- ☐ They are relatively prime.

1.3 Mrs. Smith has bolts of material that have lengths of 42, 63, and 105 inches. She needs to cut the fabric into equal length pieces with none left over. What is the greatest length of fabric pieces she can cut?
- ☐ 3 inches
- ☐ 7 inches
- ☐ 21 inches
- ☐ 42 inches

1.4 One number has a prime factorization of $2^3 \cdot 3^2$, and another number has a prime factorization of $2^2 \cdot 3^3$. Which of the following expressions would equal the greatest common factor of these two numbers?
- ☐ $2^5 \cdot 3^5$
- ☐ $2^3 \cdot 3^3$
- ☐ $2^2 \cdot 3^2$
- ☐ $2 \cdot 3$

1.5 What is the prime factorization of $81m^2x^3$?
- ☐ $9^2 \cdot m^2 \cdot x^3$
- ☐ $3^3 \cdot m \cdot x$
- ☐ $3^3 \cdot m^2 \cdot x^3$
- ☐ $3^4 \cdot m^2 \cdot x^3$

1.6 Find the GCF of 75, 100, and 175.
- ☐ 75
- ☐ 20
- ☐ 25
- ☐ 5

1.7 Find the GCF of n^3t^2 and nt^4.
- ☐ n^4t^6
- ☐ n^3t^6
- ☐ n^3t^4
- ☐ nt^2

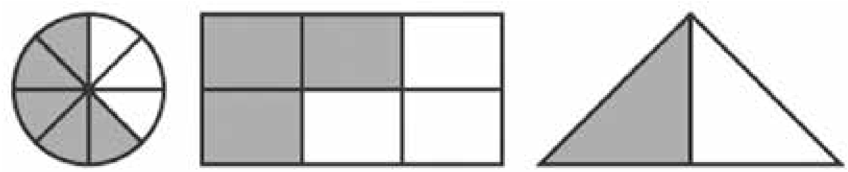
1.8 Which of the following statements is true about 63 and 20?
- ☐ They are relatively prime.
- ☐ They each have a factor of 4.
- ☐ They have more than one common factor.
- ☐ They are both prime numbers.

1.9 The greatest common factor of $60w^5y^3$ and $78wy^2$ is _____.
- ☐ $2wy^2$
- ☐ $6wy^2$
- ☐ $6w^5y^3$
- ☐ $13w^5y^3$

1.10 Find the GCF of $22ab^2c^2$ and $40a^3c^2$.
- ☐ $4a^3b^2c^2$
- ☐ $2ab^2c^2$
- ☐ $4ac^2$
- ☐ $2ac^2$

Simplifying Fractions

Fractions are used to represent parts of a whole. We can actually use shapes to represent fractions. A shape can be cut into any number of pieces that are the same size and shape. Then, some of the pieces can be shaded in. The total number of pieces represents the denominator, or the bottom number in the fraction. The number of shaded pieces represents the numerator, or the top number in the fraction. See if you can determine what fractions each of the following shapes represents.



Objectives

- Reduce positive and negative fractions.
- Reduce fractions with variables.

Vocabulary

denominator—the bottom part of a fraction; represents the whole

equivalent fractions—fractions that may be expressed differently, but still have the same value

greatest common factor—the largest number that divides evenly into two or more given numbers

improper fraction—a fraction with a numerator that is larger than or equal to the denominator

mixed number—consists of an integer and a proper fraction

numerator—the top part of a fraction; represents part of the whole

proper fraction—a fraction with a numerator that is less than the denominator

Reducing Fractions

Let's see how you did with the shapes in the introduction. In the circle, five of eight sections were shaded. So, the circle represents the fraction $\frac{5}{8}$. In the rectangle, three of six sections were shaded, or $\frac{3}{6}$. And, in the triangle, one of two sections was shaded, or $\frac{1}{2}$. Each of these fractions is a *proper fraction*, which means that the *numerator* is smaller than the *denominator*.

Vocabulary! When the numerator is larger than the denominator, the fraction is an *improper fraction*. Improper fractions can be converted to *mixed numbers* using division. For example, $\frac{3}{2}$ can also be written as $1\frac{1}{2}$.

Fractions can be either positive or negative. That's because fractions are a type of rational number. Remember that rational numbers include any number that can be written as the ratio of two integers. As you'll see, many of the rules that apply to integers also apply to fractions!

When working with fractions, you should express the answer in reduced, or simplified, form. In reduced form, the numerator and denominator don't have any common factors. To reduce a fraction, divide the numerator and the denominator by the same factor. The GCF, or *greatest common factor*, of the numerator and denominator is the largest factor that goes into both.

This might help! If you are asked to simplify an answer to lowest terms, you are being asked to reduce the fraction. If the numerator and denominator are relatively prime, or have no prime factors in common, then the fraction is already in reduced form.

For example, to reduce $\frac{18}{30}$, find the GCF of 18 and 30.

$$18 = 2 \cdot 3^2$$

$$30 = 2 \cdot 3 \cdot 5$$

So, the GCF of 18 and 30 is $2 \cdot 3$, or 6. To reduce the fraction completely, divide both the numerator and denominator by 6.

$$\frac{18 \div 6}{30 \div 6} = \frac{3}{5}$$

Two fractions that have the same value are called equivalent fractions. $\frac{18}{30}$ and $\frac{3}{5}$ are a pair of equivalent fractions.

Example:

- Simplify $-\frac{63}{225}$ to its lowest terms.

Solution:

- Find the GCF of 63 and 225.
- $63 = 3^2 \cdot 7$
- $225 = 3^2 \cdot 5^2$
- The greatest common factor is 3^2, or 9. So, divide both the numerator and denominator by 9.

Make note! When reducing positive and negative fractions, follow the rules for dividing integers. We can do that because a fraction bar means division. The quotient of two numbers with the same sign is positive. The quotient of two numbers with different signs is negative. If the negative sign is in front of the entire fraction, as in this example, then the equivalent fraction is also negative.

- $\frac{63 \div 9}{225 \div 9} = \frac{7}{25}$
- So, $-\frac{63}{225}$ simplifies to $-\frac{7}{25}$.

There's another way we can think about reducing, or simplifying, fractions. Look back at the last example. Once we had the prime factorization of each number, we could have canceled the common factors. Notice that if we cancel the common factor, 3^2, from 63, we're left with 7. Canceling 3^2 from 225 leaves us with 5^2, or 25. Either way we approach the problem, using division by the GCF or cancellation of the common factors, we're left with the same reduced fraction. The key is to divide by the same number, or cancel the same factors, in both the numerator and denominator.

The cancellation method, however, is extremely useful when we have fractions with variables. Take a look.

Example:

- Reduce the fraction $\frac{x^4yz^2}{x^3y^3z^2}$.

Solution:

- Find the GCF of the numerator and denominator. Variables are already expressed as a prime factorization. The two terms have x^3yz^2 in common. If you cancel x^3yz^2 from the numerator, you're only left with one x. If you cancel x^3yz^2 from the denominator, you're left with two y's, or y^2. So, the fraction reduces to $\frac{x}{y^2}$.
- The factors can also be written out and cancelled.

Canceling Factors

$$\frac{x^4yz^2}{x^3y^3z^2} = \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot\cancel{y}\cdot\cancel{z}\cdot\cancel{z}}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{y}\cdot y\cdot y\cdot\cancel{z}\cdot\cancel{z}} = \frac{x}{y^2}$$

- Let's look at one more example.

Example:

- Simplify $\frac{54m^4n^5}{-36m^2n^3}$.

Solution:

- Start with the constants, and then work with the variables.
- First, find the prime factorizations of 54 and 36.
  - $54 = 2 \cdot 3^3$
  - $36 = 2^2 \cdot 3^2$
- The GCF of 54 and 36 is $2 \cdot 3^2$, or 18. So, cancel those factors out, or divide both the numerator and denominator by 18. Either way, in the numerator, there will be a factor of 3 left, and in the denominator, there will be a factor of 2 left.
- $-\frac{54m^4n^5}{36m^2n^3} = -\frac{\cancel{2}\cdot 3\cdot\cancel{3}\cdot\cancel{3}\cdot m^4n^5}{\cancel{2}\cdot 2\cdot\cancel{3}\cdot\cancel{3}\cdot m^2n^3} = -\frac{3m^4n^5}{2m^2n^3}$
- The variables have a GCF of m^2n^3. Cancel those factors out of each part of the fraction.
- $-\frac{3m^4n^5}{2m^2n^3} = -\frac{3\cdot\cancel{m}\cdot\cancel{m}\cdot m\cdot m\cdot n\cdot n\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}}{2\cdot\cancel{m}\cdot\cancel{m}\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}} = -\frac{3m^2n^2}{2}$
- The simplified fraction is $-\frac{3m^2n^2}{2}$.

Make note! Notice that only the denominator of the fraction is negative. Remember that with integers, a positive number divided by a negative number is negative. So, in reduced form, the fraction will be negative.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- Fractions represent part of a whole.
- Two fractions that have the same value are equivalent fractions.
- Fractions can be simplified by dividing the numerator and denominator by the GCF, or by canceling out the common factors.
- To determine the sign of the fraction, follow the rules for dividing integers.

Complete the following activities.

1.11 If both the numerator and denominator of $\frac{48}{64}$ are divided by 8, the fraction will be in lowest terms.

- ○ True
- ○ False

1.12 If the numerator of a fraction is positive and the denominator negative, then its equivalent reduced fraction will also be negative.

- ○ True
- ○ False

1.13 $\frac{-156}{396}$ and $-\frac{26}{33}$ are equivalent fractions.

- ○ True
- ○ False

1.14 What is the GCF of the numerator and denominator in the following fraction? $-\frac{99}{176}$

- ☐ 3
- ☐ 11
- ☐ 9
- ☐ They are relatively prime.

1.15 Reduce $\frac{-56}{-70}$ to lowest terms.

- ☐ $\frac{8}{10}$
- ☐ $-\frac{4}{5}$
- ☐ $-\frac{2}{3}$
- ☐ $\frac{4}{5}$

1.16 Reduce $\frac{-34}{87}$ to lowest terms.

- ☐ $-\frac{2}{5}$
- ☐ $-\frac{11}{29}$
- ☐ $-\frac{1}{3}$
- ☐ It is already reduced.

1.17 What is the GCF of the numerator and denominator in the following fraction? $\frac{21m^2n}{28mn^2}$

- ☐ $7m^2n^2$
- ☐ $7mn$
- ☐ $21m^3n^3$
- ☐ mn

1.18 Reduce the following fraction: $\frac{-32abc}{64a^2bc^3}$.

- ☐ $-\frac{1}{2ac^2}$
- ☐ $-\frac{16ac}{32ac^2}$
- ☐ $\frac{1ac^2}{2}$
- ☐ $-\frac{32}{64ac^2}$

SELF TEST 1: Number Theory

Complete the following activities (6 points, each numbered activity).

1.01 The LCM of 50, 120, and 225 is 5.
- ○ True
- ○ False

1.02 The greatest common factor of $56f^3g^2$ and $70fg^3$ is $7fg^3$.
- ○ True
- ○ False

1.03 The LCD of $\frac{6y}{5x}$ and $\frac{8}{12x}$ is $60x$.
- ○ True
- ○ False

1.04 The fractions $\frac{-42}{-45}$ and $\frac{14}{15}$ are equivalent fractions.
- ○ True
- ○ False

1.05 The numbers 77 and 791 are relatively prime.
- ○ True
- ○ False

1.06 Which of the following expressions is the prime factorization of 360?
- ☐ $2^2 \cdot 3^2 \cdot 5$
- ☐ $2^3 \cdot 3^2 \cdot 5$
- ☐ $2^4 \cdot 3 \cdot 5$
- ☐ $2^3 \cdot 5 \cdot 9$

1.07 The manager of a store wants to have a sales promotion where she gives prizes to the first several people through the door. The manager wants each person to receive the exact same items and in the same amount. She also wants to distribute all of the prizes. The manager has 240 pins, 360 ornaments, and 540 mugs to distribute. How many people will get gifts?
- ☐ 240
- ☐ 1,140
- ☐ 15
- ☐ 60

1.08 Which of the following is the least common multiple of 24 and 36?
- ☐ 6
- ☐ 12
- ☐ 72
- ☐ 864

1.09 Katy wants to reduce $\frac{180}{315}$, but she wants to do it in one step. Which of the following is the greatest common factor that she should use to reduce this fraction?

☐ 5 ☐ 45 ☐ 9 ☐ 30

1.010 Which of the following expressions represents the GCF of $91x^2y$ and $104xy^3$?

☐ $13xy$ ☐ $728x^2y^3$ ☐ $13x^2y$ ☐ $728xy$

1.011 Which of the following expressions represents the LCM of $91x^2y$ and $104xy^3$?

☐ $13xy$ ☐ $728x^2y^3$ ☐ $13x^2y$ ☐ $728xy$

1.012 Which of the following prime factorizations represents the greatest common factor of 162, 378, and 414?

☐ $2 \cdot 3^3$ ☐ $2 \cdot 3^2 \cdot 7 \cdot 23$

☐ $2 \cdot 3^4 \cdot 7 \cdot 23$ ☐ $2 \cdot 3^2$

1.013 Match each pair of equivalent fractions.

________	$\frac{-4a}{8ab}$	-1/2b
________	$\frac{-9b^3}{-3a^2b}$	$3b^2/1a^2$
________	$\frac{3a^4}{1a^3b^2}$	$3a/b^2$
________	$\frac{5a^4b}{-10a^2}$	$-1a^2b/2$

1.014 When the following fraction is reduced, what will be the exponent on the *m*? $\frac{27mn^3}{51m^6n}$

☐ 3 ☐ 4 ☐ 5 ☐ 6

1.015 What is the LCD of $\frac{3k}{40}$ and $\frac{k}{18}$?

☐ 360 ☐ 720 ☐ $3k$ ☐ $3k^2$

1.016 The least common multiple of $54c^2de^3$ and $38c^4$ is _____.

☐ $342c^2$ ☐ $1{,}026c^4de^3$ ☐ $342c^4de^3$ ☐ $1{,}026c^4$

77/96 **SCORE** ____________ **TEACHER** ____________ ____________
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