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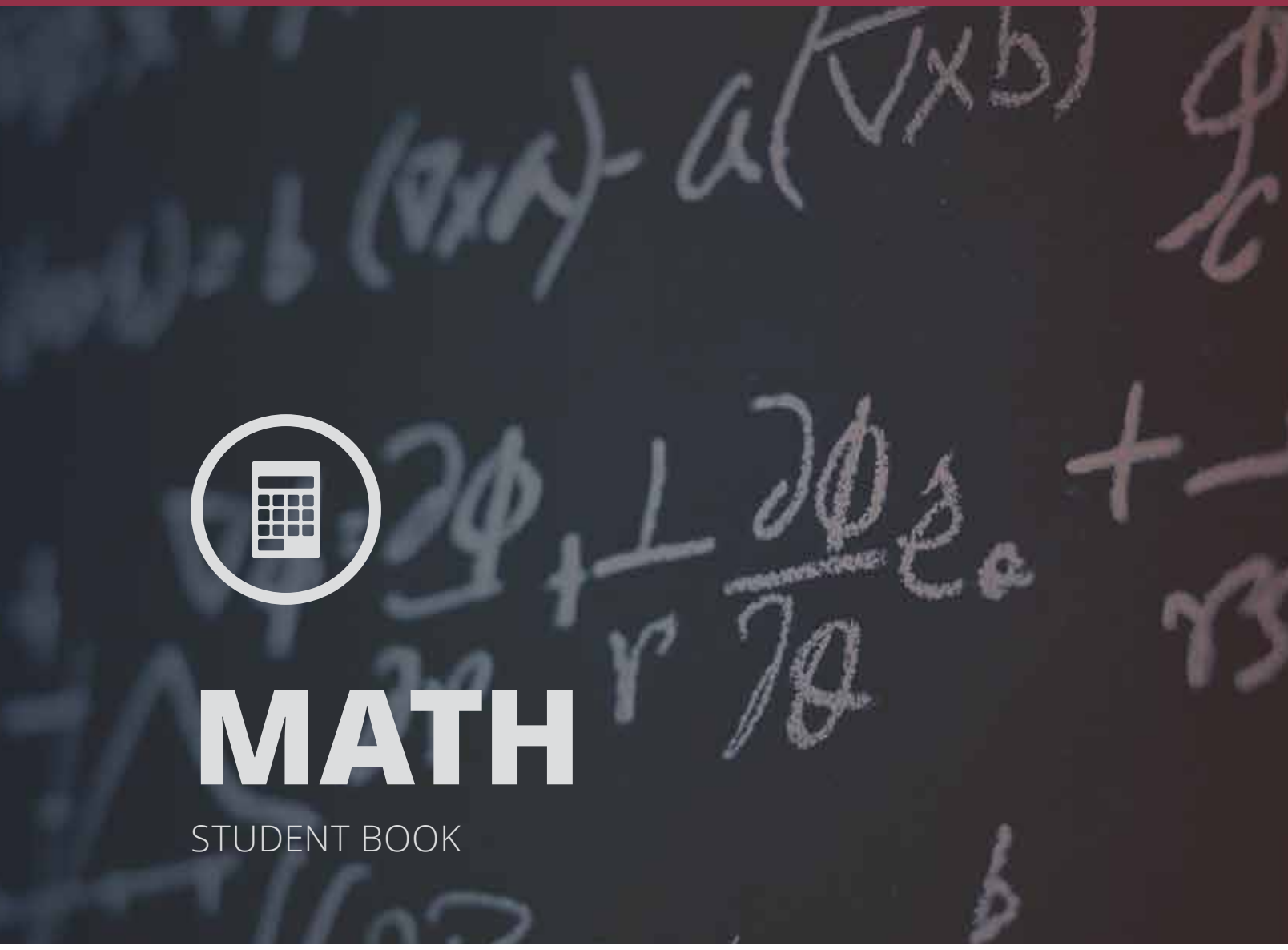

MATH

STUDENT BOOK

7th Grade | Unit 10

Alpha Omega
PUBLICATIONS

Math 710

Surface Area and Volume

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Surface Area and Volume

Introduction

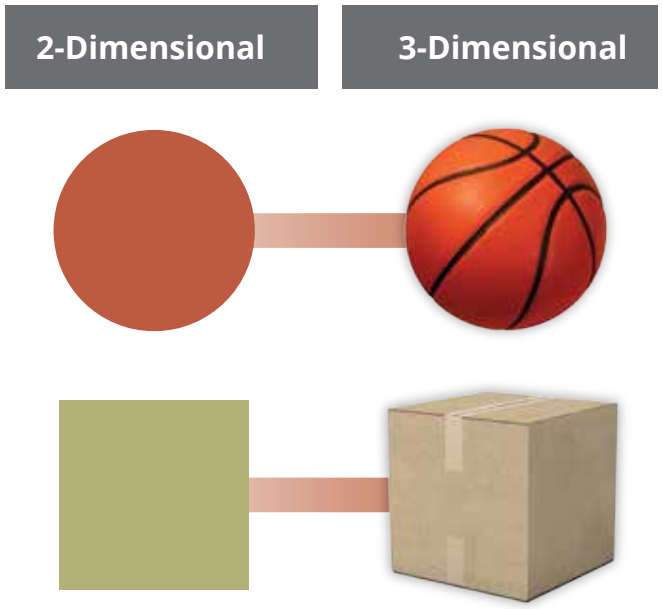
In this unit, students will explore solid, or three-dimensional, figures. They will learn how to classify and identify these solids, as well as represent them two-dimensionally. They will also learn to determine the surface area and volume of many solids, including rectangular prisms, triangular prisms, and cylinders.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Classify, identify, and draw the net of solid figures.
- Define surface area and volume.
- Find the surface area and volume of solid figures using their nets.
- Apply the surface area formulas for rectangular prisms, triangular prisms and cylinders.
- Apply the volume formulas for rectangular prisms, triangular prisms, and cylinders.
- Determine the effects of dimension changes on the surface area and volume of solid figures.

1. Solids



Classifying and Identifying Solids

In this lesson, you'll begin exploring three-dimensional figures and their characteristics.

Objectives

- Classify and identify solid figures.

Vocabulary

apex—the point at the tip of a cone or pyramid

base—a special face of a solid figure

cone—a three-dimensional figure with a circular base

cube—a three-dimensional figure made of six congruent squares

cylinder—a three-dimensional figure with two parallel, congruent, circular bases and a curved surface

edge—a line segment where two faces meet

face—a plane figure that is one side of a solid figure

lateral face—any face that is not a base

lateral surface—any surface that is not a base

plane figure—geometric figure with two dimensions (2D)

prism—three-dimensional figure with two parallel, congruent, polygonal faces and parallelograms for all other faces

solid figure—geometric figure with three dimensions (3D)

vertex—point where three or more edges meet

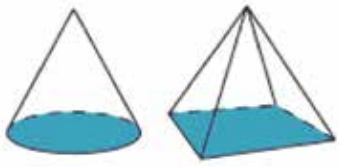
You already know a lot about two-dimensional figures, or *plane figures*. Plane figures have height and width and include quadrilaterals, triangles, circles, and other such shapes. Now you're going to start looking at geometric figures that have a third dimension—depth. These three-dimensional figures are called *solid figures* (or just solids).

Every solid figure has a *base*, which is usually the bottom of it. On a plane figure, the base of the figure is just a line segment. But on a solid figure, the base of the figure is an entire two-dimensional shape, or plane figure. Bases are used to help name the figure. If the solid has another side that is congruent *and* parallel to the base, then the figure actually has two bases. Take a look at some solids that have two bases. The bases are shaded so that you can easily

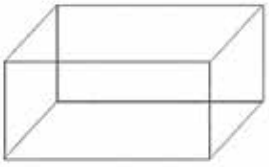
see them. Notice that each base is a flat, plane figure.

Vocabulary! Remember that to be congruent, two figures have to have the same size and shape. Being parallel means that the figures have no points in common.

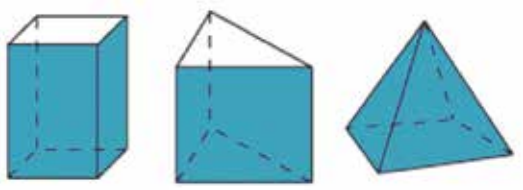
Here are some examples of solid figures that have only one base.



All of the sides of a solid figure that are plane figures, including the bases, are called *faces*. Each line segment where two faces meet is called an *edge*. Each point where three or more edges meet is called a *vertex*. The solid figure in the following illustration has six faces, twelve edges, and eight vertices.

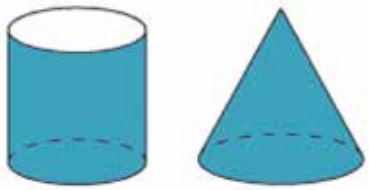


The faces that aren't bases of the solid are called *lateral faces*. The lateral faces are shaded in the following solid figures. Take note that the base on the bottom of each figure is not considered a lateral face; just the sides are lateral faces.



Remember that plane figures are flat. Some solid figures have sides that are not flat, so they're not considered faces. These sides are called *lateral surfaces*, rather than lateral faces. The lateral surfaces are shaded in the following two solids. As in the previous examples, remember that the bases of each figure are not considered part of the lateral surface; only the side of the figure is.

Vocabulary! In geometry, the word lateral always refers to the sides of a solid. It doesn't include the base(s) of a solid. So whenever you see the word lateral, remember to just include the sides.



What do you notice about the figures that have lateral faces compared to the figures that have lateral *surfaces*? The bases of the solids that have lateral faces are polygons, and the bases of the solids that have lateral surfaces are circles! That's because if the base of the solid is circular, then the sides have to be curved in order to accommodate the shape of the base.

Example:

- Answer the following questions based on the following solid figure.

Solution:

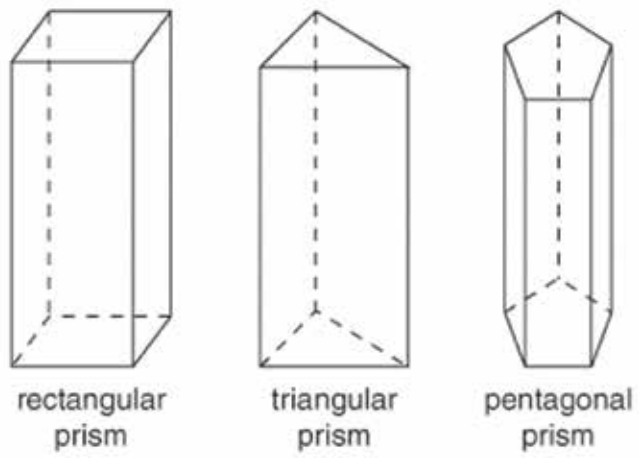
- Question: Does the solid have one or two bases?
  Answer: The base, or bottom, of the solid is a pentagon. Since there is another congruent pentagon that is parallel to the bottom of the solid, it has two bases.
- Question: Does the solid have lateral faces or lateral surfaces?
  Answer: The sides of the solid are plane figures (flat), so the sides are lateral faces.
- Question: How many faces does the solid have?
  Answer: There are seven faces (including the bases).
- Question: How many lateral faces does the solid have?
  Answer: There are five lateral faces (not including the bases).
- Question: How many edges does the solid have?
  Answer: There are fifteen edges (five vertical edges and five on each base).
- Question: How many vertices does the solid have?
  Answer: There are ten vertices (five on each base).

Now take a look at the names of some of the most common solid figures.

Prisms

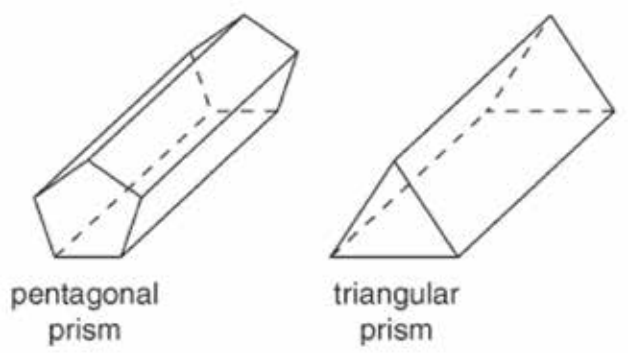
A *prism* is a type of solid figure that has two bases. The bases of a prism are always polygons. The lateral faces of a prism are always parallelograms. Prisms are named by the shape of their bases. For example, a prism that has two congruent rectangular bases is called a rectangular prism. A prism that has two congruent triangular bases is called a triangular prism. And a prism that has two congruent pentagonal bases is called a pentagonal prism. Here are some examples.

This might help! On all of the solid figures in this unit, dashed lines are used to represent the edges that can't actually be seen from the given point of view. Solid lines are used for the edges that can be seen from the given point of view.



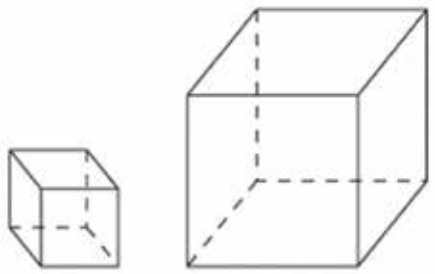
Sometimes a prism may be lying on its side. However, the name of the prism remains the same because the base remains the same—even though it technically isn't the bottom anymore. That's because the

definition of a prism is that there are always two congruent polygonal bases, and the rest of the faces are *always* parallelograms. So a triangular or pentagonal prism may be lying on its side, but the triangles and pentagons are still considered the bases of the figure.



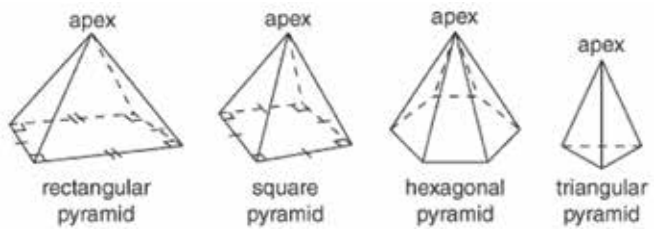
There is one special type of rectangular prism that you need to look at. A *cube* is a prism that has six congruent, square faces.

Make note! The cube is an important solid figure in geometry. It can be used to find the volume (or capacity) of solid figures.



Pyramids

A *pyramid* is similar to a prism in that it has a polygon-shaped base. However, instead of two bases, a pyramid only has one base. The lateral sides of a pyramid are always triangles, which meet at a single point, called an *apex*. Like prisms, pyramids are named based on the shape of their base.



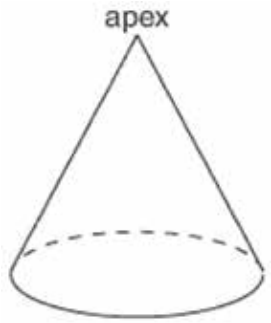
Keep in mind! If there are no measurements given, how can you tell whether the base of a prism or pyramid is a rectangle or a square? Often, the geometric markings that you've learned will be used. Remember that tick marks are used to show which sides have the same length, and a square in the corner of an angle represents a right angle.

Cylinders

A *cylinder* has two bases like a prism, but its bases are always circles, rather than polygons. The side of a cylinder is curved, rather than flat, so it is called a lateral surface. In fact, the cylinder only has two faces—the bases. As you can see from the following figures, cylinders can also lay on their sides.

Cones

A *cone* is similar to a cylinder in that it has a circular base and the side is curved rather than flat. It's different, though, in that it only has *one* base. The lateral surface comes to a point at the top of the cone. So like a pyramid, the cone has an apex.



Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- Solid figures are three-dimensional shapes—having height, width, and depth.
- Solid figures can have one or two bases. Pyramids and cones have one base. Prisms and cylinders have two bases.
- If the base of the solid is a polygon, the base is used to name the figure.
- Lateral faces or surfaces are the sides of a solid figure.

Complete the following activities.

1.1 The only solid figure that has two circular bases is a _____ .

☐ pyramid ☐ cone ☐ cylinder ☐ prism

1.2 A _____ does not have an apex.

☐ cone ☐ prism ☐ pyramid

1.3 A _____ prism has two congruent, parallel bases and three lateral sides.

☐ triangular ☐ rectangular ☐ pentagonal

1.4 A prism that has six congruent, square faces is called _____ .

☐ a cube ☐ an apex ☐ a pyramid

1.5 A triangular pyramid has _____ faces.

☐ two ☐ three ☐ four ☐ five

1.6 A cone is composed of two congruent bases and a lateral surface.

○ True
○ False

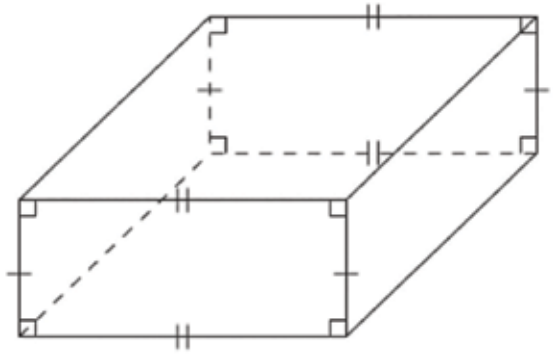
1.7 A cylinder has no lateral faces.

- ○ True
- ○ False

1.8 All of the following solid figures *except* _____ have two congruent, parallel bases.

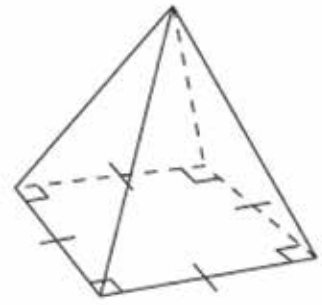
- ☐ rectangular pyramid
- ☐ heptagonal prism
- ☐ cylinder
- ☐ cube

1.9 What is the name of the following solid figure?



- ☐ square prism
- ☐ rectangular prism
- ☐ cube
- ☐ rectangular pyramid

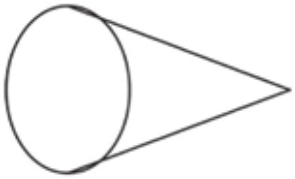
1.10 What is the name of the following solid figure?



- ☐ square prism
- ☐ cube
- ☐ square pyramid
- ☐ rectangular prism

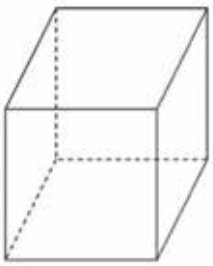
Identify the figures below.

1.11 _____

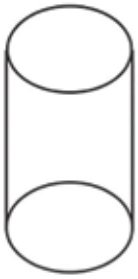


1.12 _____

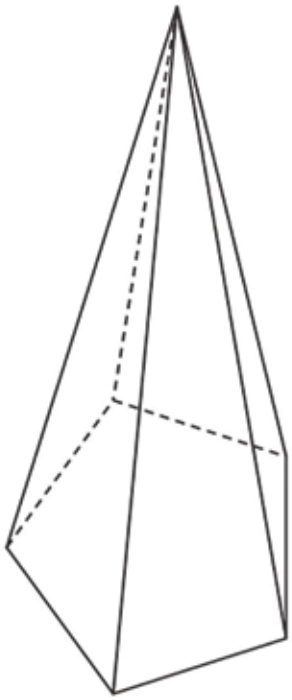
1.13 All edges are equal. _____



1.14 _____



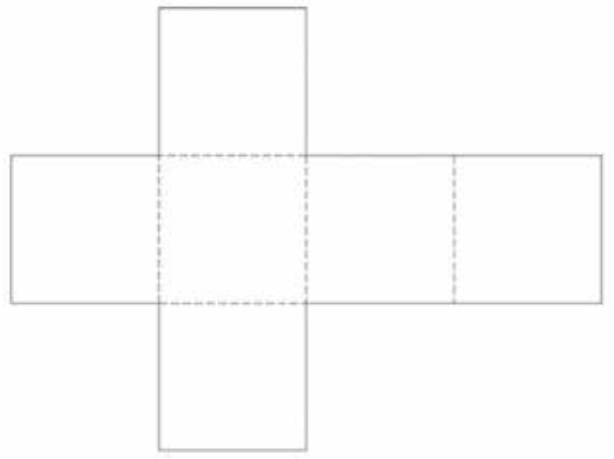
1.15 _____



Nets

For the following activity, you'll need a sheet of graphs paper, scissors, and tape:

Draw a figure on the graph paper like the one to the right. All of the individual squares must be the same size.



Cut out the t-shaped figure. You should end up with a single T-shaped piece.

Fold along all the dotted lines inside the figure.

Assemble the figure into a cube and secure the edges with tape.

You have now formed a three-dimensional figure using two-dimensional shapes! In this lesson, you'll continue to learn how prisms, pyramids, cones, and cylinders can be created out of plane figures.

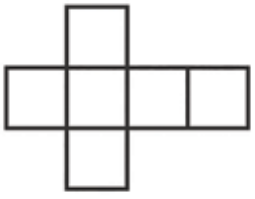
Objectives

- Identify and sketch the net of a solid figure.

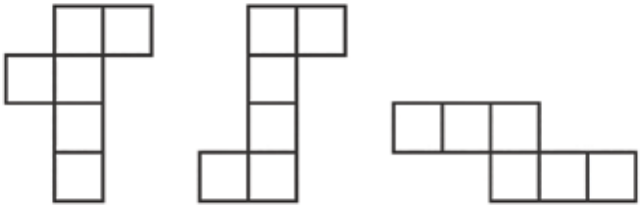
Vocabulary

net—a two-dimensional representation of a three-dimensional shape when unfolded

In the activity above, you created a three-dimensional cube from a two-dimensional cut-out. That cut-out is a two-dimensional representation of a cube and is called its *net*. The representation shows what a cube would look like if it was unfolded. Here is the net for a cube again.



You may be wondering if there is more than one possible net for a cube. The answer is yes! There are actually eleven possible two-dimensional representations of a cube. Here are a few of them:



There are a couple of things to notice about the nets of a cube. Because a cube is composed of six congruent squares, each net must have six congruent squares. However, not every representation with six congruent squares is the net of a cube. Visualizing how the representation looks as it is folded will help you determine if it is a net or not. In a net, none of the parts

Self Test 1: Solids

Complete the following activities (5 points, each numbered activity).

1.01 A cone has an apex.
- ○ True
- ○ False

1.02 The bases of a cylinder must be polygons.
- ○ True
- ○ False

1.03 A square pyramid has five faces.
- ○ True
- ○ False

1.04 The net of a cylinder has two parts.
- ○ True
- ○ False

1.05 Surface area of a solid figure can be found by multiplying the area of the base by the height of the figure.
- ○ True
- ○ False

1.06 Volume is measured in cubic units.
- ○ True
- ○ False

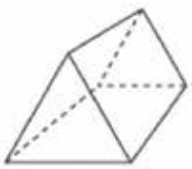
1.07 A prism _____ has two congruent, parallel bases.

☐ never ☐ sometimes ☐ always

1.08 The net of a cone is composed of a circle and a _____ .

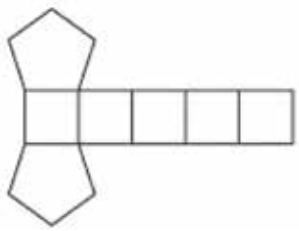
☐ triangle ☐ wedge ☐ rectangle

1.09 Identify the solid figure below.



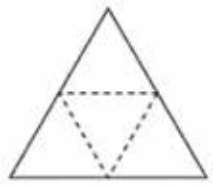
- ☐ triangular pyramid
- ☐ triangular prism
- ☐ rectangular prism
- ☐ rectangular pyramid

1.010 Which solid figure has the following net?



- ☐ pentagonal prism
- ☐ pentagonal pyramid
- ☐ hexagonal prism
- ☐ hexagonal pyramid

1.011 Which solid figure has the following net?



- ☐ triangular prism
- ☐ square pyramid
- ☐ triangular pyramid
- ☐ cone

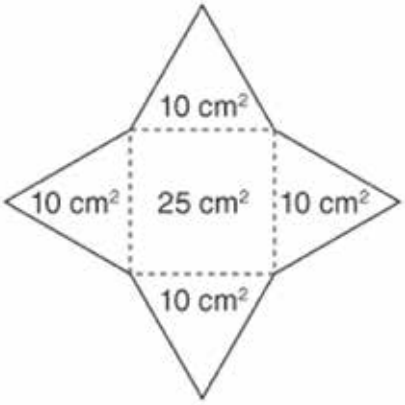
1.012 What is the surface area of a cylinder that has the following measurements?
area of each base: 50.24 ft^2
area of lateral surface: 75.36 ft^2
height of cylinder: 3 ft

- ☐ 125.6 ft^2
- ☐ 150.72 ft^2
- ☐ 128.6 ft^2
- ☐ 175.84 ft^2

1.013 What is the volume of a cylinder that has the following measurements?
area of each base: 50.24 ft^2
area of lateral surface: 75.36 ft^2
height of cylinder: 3 ft

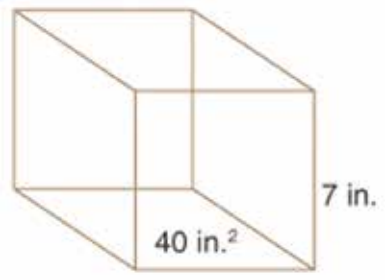
- ☐ 150.72 ft^3
- ☐ 226.08 ft^3
- ☐ 128.6 ft^3
- ☐ 175.84 ft^3

1.014 Find the surface area of the solid figure represented by the given net.



- ☐ 250 cm^2
- ☐ 65 cm^2
- ☐ 90 cm^2
- ☐ 40 cm^2

1.015 Find the volume of the solid figure shown below. The area of the base is 40 square inches, and the height of the figure is 7 inches.



- ☐ 47 $in.^3$
- ☐ 108 $in.^3$
- ☐ 68 $in.^3$
- ☐ 280 $in.^3$

1.016 The area of the base of a prism is 19 square feet. What is the volume of the prism if the height is 3 feet?

1.017 A cube has a side that is 6 cm long. What is the surface area of the cube?

1.018 A cube has a side that is 6 cm long. What is the volume of the cube?

1.019 A cylinder has a radius of 2 inches. The rectangle portion of its net is 24 square inches. What is the surface area of the cylinder?

1.020 A cylinder has a radius of 2 inches and a height of 4 inches. What is the volume of the cylinder?

80/100 **SCORE**_____ **TEACHER**_____ _____
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