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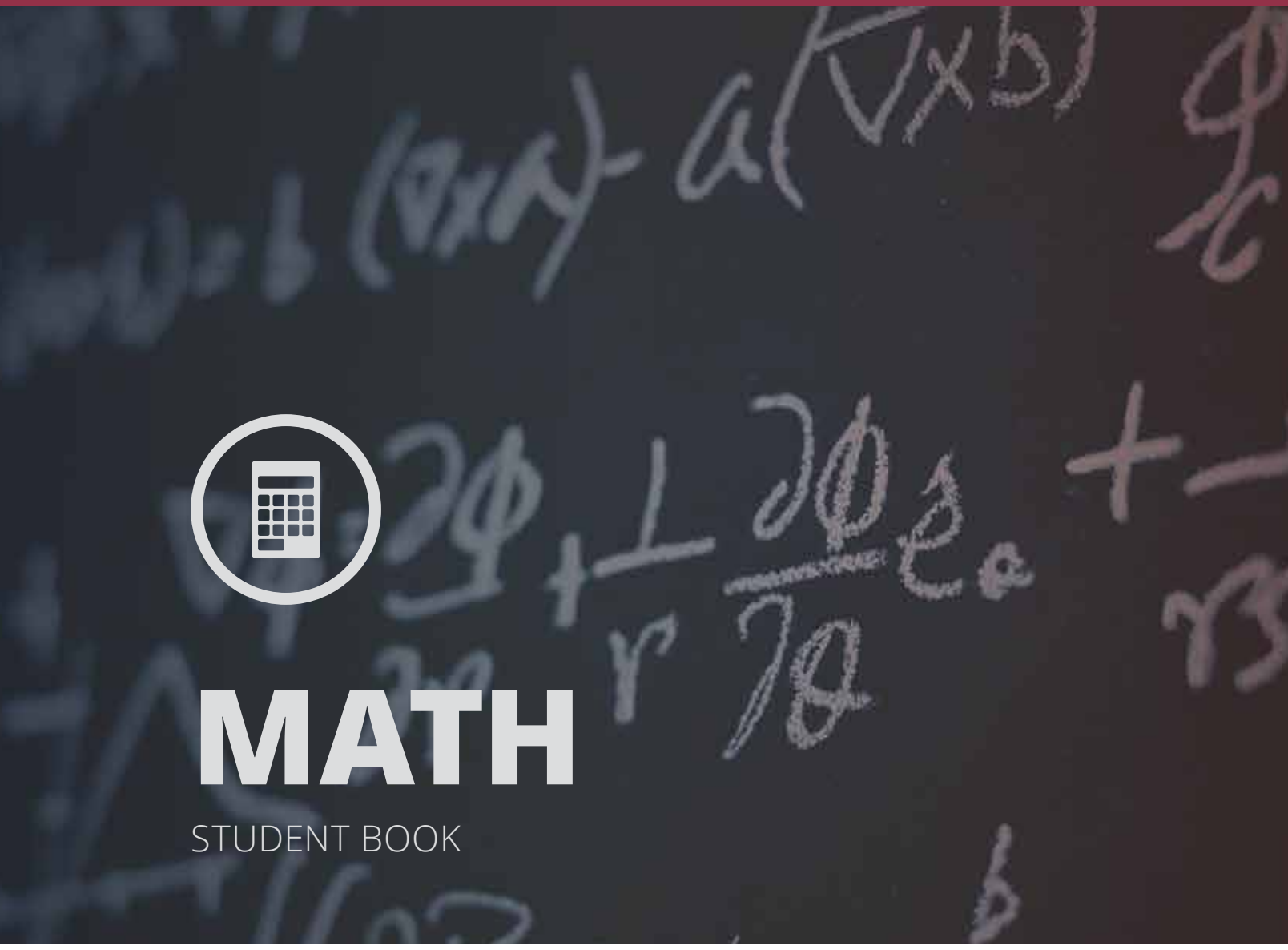

MATH

STUDENT BOOK

7th Grade | Unit 5

Alpha Omega
PUBLICATIONS

Math 705

Ratios and Proportions

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Ratios and Proportions

Introduction

In this unit, students will learn about ratios and proportions and how to apply them to various problem-solving situations. Students will use unit rates to compare different ratios and use proportions to convert between measurements. Students will also explore similar figures, congruent figures, and scale drawings, and they will use proportions to find missing lengths. Students will finish the unit by learning that fractions, decimals, and percents are different ways to represent the same value, and they will apply this knowledge to solving percent problems with proportions and equations.

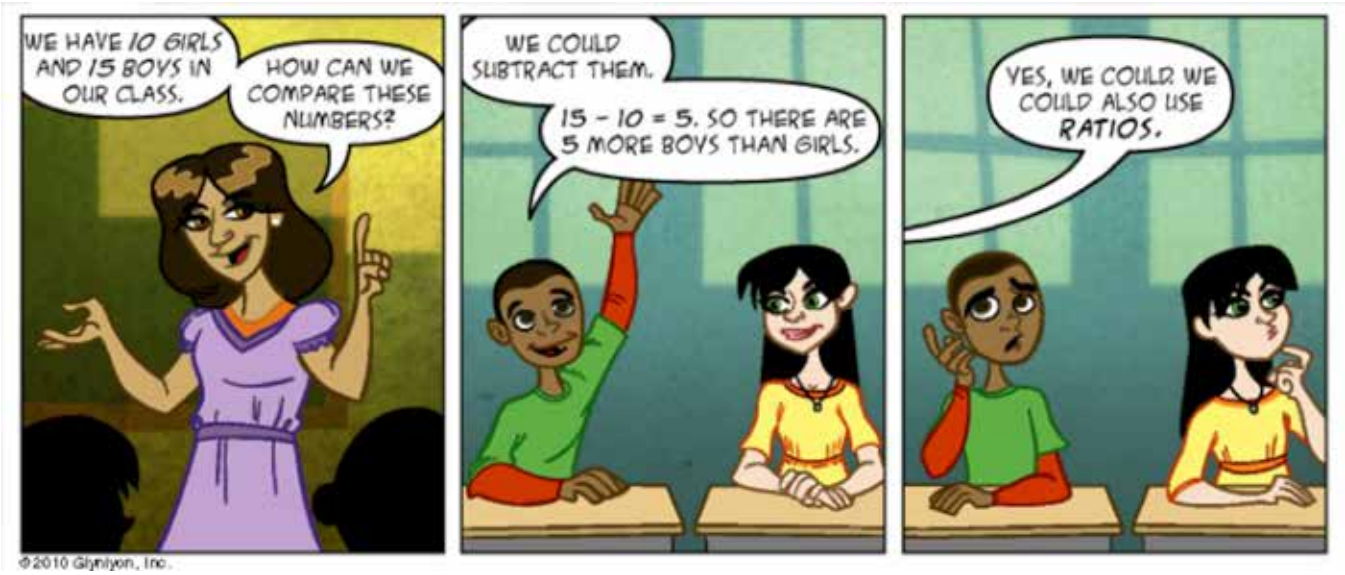
Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Write and simplify ratios and rates.
- Compare ratios using unit rates.
- Write and solve proportions.
- Convert between customary units.
- Convert between metric units.
- Recognize similar figures and work with scale drawings.
- Use a proportion to find a missing length of a similar triangle.
- Convert between fractions, decimals, and percents.
- Find the percent of a number and the percent of change.
- Solve percent problems using a proportion or an equation.

1. Ratios, Rates, and Proportions

Ratios



In this lesson, you'll learn how to use *ratios* to compare numbers. You'll also learn how to write ratios in different ways.

Objectives

- Write and simplify ratios.

Vocabulary

part-part ratio—a ratio comparing two portions of a whole quantity to each other
part-whole ratio—a ratio comparing a portion of a whole quantity to the whole quantity
ratio—a comparison of two quantities or numbers as a quotient

Writing Ratios

A ratio shows a comparison between two numbers using division. It is often written like a fraction. For instance, if there were 3 girls and 4 boys in a group, you would say that the ratio of girls to boys in the group is "3 to 4," and you would write it as $\frac{3}{4}$. A ratio can also be written using a colon. In this case, the ratio of girls to boys would be written as 3:4. The ratio of 3 girls to 4 boys means that there are $\frac{3}{4}$ as many girls as there are boys in the group.

In Mrs. Green's class there are 10 girls and 15 boys. It is important to write ratios in the correct order. If you want to show the ratio of girls to boys in Mrs. Green's class, you would write 10 to 15 because there are 10 girls to 15 boys. If you wanted to write the ratio of boys to girls, then you would write

15 to 10 since there are 15 boys to 10 girls. Look at the differences.

Girls to Boys Ratio

- 10 to 15
- 10:15
- $\frac{10}{15}$

Boys to Girls Ratio

- 15 to 10
- 15:10
- $\frac{15}{10}$

So far, you've compared one part of the class (girls) to another part (boys). Previously, you looked at ratios that compared girls to boys (10 to 15) and boys to girls (15 to 10). Both of these ratios compared a part of the whole class to another part of the whole class. This is called a *part-part ratio*:

- $\frac{\text{part}}{\text{part}} = \frac{\text{girls}}{\text{boys}} = \frac{10}{15}$
- $\frac{\text{part}}{\text{part}} = \frac{\text{boys}}{\text{girls}} = \frac{15}{10}$

You can also compare a part of the class to the whole class. This is called a *part-whole ratio*:

- $\frac{\text{part}}{\text{whole}} = \frac{\text{girls}}{\text{class}}$
- $\frac{\text{part}}{\text{whole}} = \frac{\text{boys}}{\text{class}}$

What is the ratio of girls to the total number of students in Mrs. Green's class? You know there are 10 girls and 15 boys, so there are 25 students in the class (10 + 15 = 25). So the ratio of girls to students is 10 to 25, or $\frac{10}{25}$.

How can you find the ratio of boys to students? You know that there are 15 boys in the whole class, so the ratio of boys to students is 15 to 25 or $\frac{15}{25}$.

So you have two more ratios:

- $\frac{\text{part}}{\text{whole}} = \frac{\text{girls}}{\text{class}} = \frac{10}{25}$
- $\frac{\text{part}}{\text{whole}} = \frac{\text{boys}}{\text{class}} = \frac{15}{25}$

Think about it! What if you didn't know how many boys were in the class? What if you only knew that there were 25 students in Mrs. Green's class and that 10 of them were girls? In order to find out the ratio of boys to students, you would first have to find out how many boys there were! Since you know there are 10 girls, and 25 is the whole, you could subtract the number of girls from 25 to find the number of boys (25 – 10 = 15).

Try another one.

Example:

- A bowl with 15 pieces of fruit has a mixture of apples and bananas. If there are 8 apples in the bowl, what is the ratio of bananas to total pieces of fruit?

Solution:

- You know that there are 15 pieces of fruit and 8 of them are apples. That means that there are 15 – 8 = 7 bananas. So the ratio of bananas to total pieces of fruit is 7 to 15. This can also be written as 7:15 or $\frac{7}{15}$.

Example:

- In a mixture of red and blue jellybeans, the ratio of red jellybeans to total jellybeans is 3 to 5. What is

the ratio of red jellybeans to blue jellybeans?

Solution:

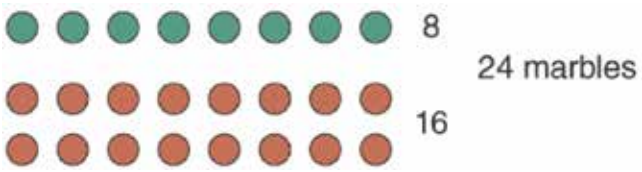
- You know that there are 3 red jellybeans out of every 5 jellybeans. That means that the other 2 (5 – 3 = 2) must be blue. So the ratio of red to blue is 3 to 2. This can also be written as 3:2 or $\frac{3}{2}$.

Now take a look at another example involving a few different ratios.

Scott has a jar full of marbles. Eight of the marbles are green and sixteen are orange. What are all of the different ratios you can write for this situation?

First, you need to find the *whole*—how many total marbles Scott has:
8 + 16 = 24

So he has 24 marbles.



Remember, you can compare part-part and part-whole, and you have three different ways to name each ratio:

Comparison	Fraction	Colons	Words
$\frac{\text{part}}{\text{part}} = \frac{\text{green}}{\text{orange}}$	$\frac{8}{16}$	8:16	8 to 16
$\frac{\text{part}}{\text{part}} = \frac{\text{orange}}{\text{green}}$	$\frac{16}{8}$	16:8	16 to 8
$\frac{\text{part}}{\text{whole}} = \frac{\text{green}}{\text{marbles}}$	$\frac{8}{24}$	8:24	8 to 24
$\frac{\text{part}}{\text{whole}} = \frac{\text{orange}}{\text{marbles}}$	$\frac{16}{24}$	16:24	16 to 24

Can you compare the whole to a part? Yes!

Comparison	Fraction	Colons	Words
$\frac{\text{whole}}{\text{part}} = \frac{\text{marbles}}{\text{green}}$	$\frac{24}{8}$	24:8	24 to 8
$\frac{\text{whole}}{\text{part}} = \frac{\text{marbles}}{\text{orange}}$	$\frac{24}{16}$	24:16	24 to 16

The ratio of 24 to 8 means that out of 24 marbles, 8 are green. If you were talking about your marble collection, you might want to first talk about how many marbles you have and then talk about how many you have of each color: "I have 24 marbles and 16 are orange."

You've probably noticed that ratios are similar to fractions. Although they do have a lot in common with fractions, they are not fractions. Look at the similarities and differences using this example of 2 pennies and 2 nickels:

Similarities:

- Both fractions and ratios can compare part-whole:
  - Fraction—A fraction of the coins are pennies.
  - Ratio—The ratio of pennies to coins is $\frac{2}{4}$, or 2 to 4.
- Both fractions and ratios represent division:
  - Fraction—The fraction $\frac{2}{4} = 2 \div 4$, or 0.5., so half the coins are pennies.
  - Ratio—The ratio $\frac{2}{4}$, or 2 to 4, means half the coins are pennies.

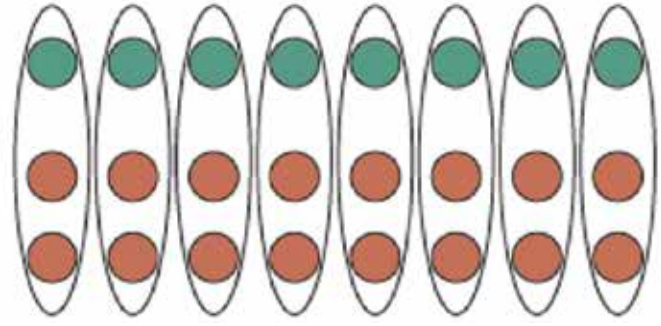
- Neither fractions nor ratios can have zero as the second number or denominator.
  - Fraction—Dividing by zero is undefined.
  - Ratio—You can't have a 2 to 0 ratio. Two pennies to zero coins isn't a comparison of two quantities.

Differences:

- Only ratios can compare part-part.
  - Fraction—The fraction $\frac{2}{2}$ equals 1. It can't compare pennies to nickels.
  - Ratio—The ratio 2 to 2 compares pennies to nickels.
- Only fractions can be written as mixed numbers.
  - Fraction—The fraction $\frac{3}{2} = 1\frac{1}{2}$.
  - Ratio—The ratio 3 to 2, or $\frac{3}{2}$, does not equal $1\frac{1}{2}$ because a mixed number is just one number, not a comparison of two numbers.

Simplifying Ratios

There is one more similarity between ratios and fractions. Ratios can be simplified in the same way as fractions. Take a look at Scott's marbles. He had 8 green marbles and 16 orange marbles, so the ratio of green to orange is 8 to 16. Just as with fractions, you look for the greatest common factor (GCF) to simplify. The GCF of 8 and 16 is 8.



$$\frac{8 \text{ green}}{16 \text{ orange}} \div \frac{8}{8} = \frac{1 \text{ green}}{2 \text{ orange}}$$

So for every green marble, there are 2 orange marbles.

Why simplify ratios? One reason is that it's usually easier to work with smaller numbers. Sometimes, simplifying a ratio will give you helpful information. This is especially true when you use ratios in proportions.

Here is an example where simplifying is useful.

Example:

- For a class party, the teacher buys 6 pizzas for 24 students. What is the ratio of pizza to students? Simplify the ratio.

Solution:

- There are 6 pizzas for 24 students, so the ratio of pizza to students is $\frac{6}{24}$. The GCF is 6, so you can divide the numerator and denominator by 6:

$$\frac{6}{24} \div \frac{6}{6} = \frac{1}{4}$$

- So there is 1 pizza for every 4 students. By simplifying the ratio, you know that each student can eat $\frac{1}{4}$ of a pizza.

Look at an example where simplifying might not be as useful.

Example:

- A recipe calls for 4 cups of flour and 6 cups of water. What is the ratio of flour to water? Simplify the ratio.

Solution:

- There are 4 cups of flour and 6 cups of water, so the ratio of flour to water is 4 to 6. The GCF is 2, so you can divide the numerator and denominator by 2:

$$\frac{4}{6} \div \frac{2}{2} = \frac{2}{3}$$

- So for every 2 cups of flour, there are 3 cups of water. This might not be that helpful to know because it changes the recipe. On the other hand, if you wanted to change the recipe, this would be very helpful.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- Ratios compare two quantities.
- A ratio can be written as a fraction, using a colon, or using the word *to*.
- Ratios are similar to fractions in several ways, but they are not fractions.
- Ratios can be simplified in the same way as fractions.

Complete the following activities.

1.1 Select all that apply. There are 11 boys and 15 girls in a class. Which of the following show the ratio of girls to boys?

☐ 15:11 ☐ 15 to 11 ☐ 11 to 15 ☐ $\frac{11}{15}$

1.2 Select all that apply. Which of the following ratios are equivalent?

☐ $\frac{3}{4}$ ☐ 3 to 4 ☐ $\frac{6}{7}$ ☐ $\frac{15}{20}$

1.3 On a true/false test, the ratio of true questions to false questions is 3 to 7. What is the ratio of true questions to total questions?

☐ 3 to 10 ☐ 3 to 7 ☐ 4 to 7 ☐ 7 to 3

1.4 On a field trip, there are 8 adults and 24 students. What is the ratio of students to the total number of people on the field trip?

☐ 8 to 24 ☐ 1 to 4 ☐ 32 to 24 ☐ 3 to 4

1.5 Simplify the ratio 36 to 45.

- ☐ $\frac{9}{5}$
- ☐ 5 to 4
- ☐ 4 to 5
- ☐ 12 to 15

1.6 Select all that apply. Several pennies are tossed in the air. Twelve land heads up and fifteen land tails up. Which of the following ratios are correct?

- ☐ 12 heads to 27 coins
- ☐ 5 tails to 9 coins
- ☐ 15 tails to 12 heads
- ☐ 12 heads to 15 tails

1.7 Select all that apply. Which of the following are true?

- ☐ Fractions and ratios cannot have zero in the denominator.
- ☐ Some fractions and ratios can be written as mixed numbers.
- ☐ Fractions and ratios are different names for the same thing.
- ☐ Fractions and ratios can be simplified the same way.

1.8 Select all that apply. Which of the following represent the same ratio?

- ☐ 2 to 3
- ☐ 2:5
- ☐ 3 to 2
- ☐ $\frac{2}{3}$

1.9 In a library where the ratio of men to women is 1 to 1, _____ of the total members are men.

- ☐ 2
- ☐ $\frac{1}{1}$
- ☐ $\frac{1}{2}$
- ☐ 3

1.10 Simplify the ratio $\frac{24}{36}$.

- ☐ 2 to 3
- ☐ 24 to 36
- ☐ 4 to 6
- ☐ 3 to 8

Complete the following activities.

1.11 An animal shelter has 25 dogs and 17 cats. What is the ratio of dogs to cats?

1.12 An animal shelter has 25 dogs and 17 cats. What is the ratio of cats to dogs?

1.13 An animal shelter has 25 dogs and 17 cats. What is the ratio of dogs to all animals?

1.14 An animal shelter has 25 dogs and 17 cats. What is the ratio of cats to all animals?

1.15 An animal shelter has 25 dogs and 17 cats. What is the ratio of all animals to dogs?

Rates

So which price is better? How can you tell? In this lesson, you'll learn how to calculate *rates*, such as price, and *unit rates*, which can help you compare prices and other rates.

Objectives

- Write and simplify rates.
- Compare ratios using unit rates.

Vocabulary

rate—a type of ratio that compares two different kinds of quantities or numbers

speed—an average rate comparing distance to time

unit rate—a rate with a denominator of 1; a rate that shows an amount of something compared to 1 of something else

Writing Rates

So which price is better? To decide, you need to find out how much one can of soda costs at each price. The prices Ondi and Carlton talked about are an example of rates. A rate is a ratio that compares two numbers with different units. Ondi and Carlton are comparing dollars to cans of soda:

Keep in mind! Remember, ratios compare one quantity to another and can be written in three different ways:
— using the word "to," as in 3 to 4
— using a colon, as in 3:4
— as a fraction, as in $\frac{3}{4}$

$$\frac{\$}{\text{cans}} = \frac{\$6.00}{24 \text{ cans}}$$

$$\frac{\$}{\text{cans}} = \frac{\$7.20}{30 \text{ cans}}$$

Self Test 1: Ratios, Rates, and Proportions

Complete the following activities (5 points, each numbered activity)

1.01 If x is to 9 as 8 is to 12, then x is equal to _____ .

1.02 If $\frac{3}{4} = \frac{x}{8}$, then x is equal to _____ .

1.03 The ratio of girls to boys is 3 to 4. If there are 48 boys, how many girls are there?

☐ 24 ☐ 40 ☐ 47 ☐ 36

1.04 What speed is a car traveling if it goes 240 miles in 4 hours?

☐ 60 miles per hour ☐ 80 miles per hour

☐ 40 miles per hour ☐ 30 miles per hour

1.05 All of the following ratios are equivalent *except* _____.

☐ 8 to 12 ☐ $\frac{15}{10}$ ☐ $\frac{2}{3}$ ☐ 6:9

1.06 If a baseball team's ratio of wins to losses is 3 to 4, what is the ratio of wins to games played? (There are no ties).

☐ 4 to 3 ☐ 3 to 7 ☐ 4 to 7 ☐ 3 to 4

1.07 Your heart beats 320 times in 5 minutes. What is your heart rate?

☐ 64 beats per minute ☐ 6.4 beats per minute

☐ 32 beats per minute ☐ 60 beats per minute

1.08 The cross products of a proportion are never equal.

○ True
○ False

1.09 Select all that apply. Which of the following are proportions?

☐ $\frac{25}{10} = \frac{5}{2}$ ☐ $\frac{3}{4} = \frac{8}{10}$ ☐ $\frac{2}{8} = \frac{3}{12}$ ☐ $\frac{3}{4} = \frac{15}{20}$

1.010 Select all that apply. Which of the following are equivalent ratios?

☐ $\frac{14}{21}$ ☐ $\frac{12}{16}$ ☐ $\frac{8}{12}$ ☐ $\frac{10}{15}$

1.011 Select all that apply. There is a ratio of 3 boys to 4 girls in a class. Which of the following are correct ratios for the class?

- ☐ 3 boys to 7 students
- ☐ 3 boys to 7 girls
- ☐ 4 girls to 3 boys
- ☐ 4 girls to 7 students

1.012 Which rate is the lowest price?

- ☐ $6.20 for 4
- ☐ $5.50 for 5
- ☐ $5.00 for 4
- ☐ $1.15 each

1.013 The price of bananas is $4.80 for 4 pounds. What is the price as a unit rate?

- ☐ $\frac{\$1.20}{4\text{ lb}}$
- ☐ $\frac{\$4.80}{1\text{ lb}}$
- ☐ $\frac{\$1.60}{1\text{ lb}}$
- ☐ $\frac{\$1.20}{1\text{ lb}}$

1.014 You can solve the following proportion by cross multiplying. What will the equation be after you cross multiply? $\frac{7}{8} = \frac{x}{12}$

- ☐ $96 = 7x$
- ☐ $12x = 56$
- ☐ $7x = 96$
- ☐ $84 = 8x$

1.015 In a library where the ratio of men to women is 3 to 1, _____ of the members are men.

- ☐ 3
- ☐ $\frac{1}{3}$
- ☐ $\frac{3}{4}$
- ☐ 4

1.016 A landscape worker cares for 42 lawns in a 6-day work week. Express his daily work as a unit rate.

1.017 Solve for *x*. $\frac{8}{10} = \frac{x}{25}$

1.018 Solve for *x*. $\frac{9}{16} = \frac{27}{x}$

1.019 A day care has an infant-to-worker ratio of 7:2. If there are 21 infants, how many workers must they have?

1.020 At a swimming pool, the ratio of lifeguards to swimmers is 1 to 12. If there are 48 swimmers in the pool, how many life guards are there?

80/100 **SCORE** _____ **TEACHER** _____ _____
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