



MATH

STUDENT BOOK

▶ **6th Grade | Unit 4**

MATH 604

Fractions

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Fractions

Introduction

In this unit, you will begin studying fractions. You will learn about factoring and finding the greatest common factor so that you can simplify fractions to their lowest terms. You will learn about multiples and the least common multiple so that you can find a common denominator between two fractions. You also will learn about equivalent fractions. You will use equivalent fractions to compare and order two or three fractions. You will explore different ways to represent the same value, including proper fractions, improper fractions, mixed numbers, and decimals. Finally, you will round and estimate with fractions and mixed numbers, as well as added and subtracted time measurements.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAAC. When you have finished this LIFEPAAC, you should be able to:

- Use the divisibility rules and find the prime factorization of a number.
- List factors, find the greatest common factor (GCF), and reduce fractions.
- Convert between proper fractions, improper fractions, and mixed numbers.
- List multiples, find the least common multiple (LCM), and compare and order fractions.
- Convert between decimal numbers and fractions.
- Round and estimate with fractions.
- Add and subtract with time and find elapsed time.

1. FACTORS AND FRACTIONS

DIVISIBILITY AND PRIME FACTORIZATION

Do you remember what a prime number is? A prime number is a number that has only two **factors**, or numbers, that evenly go into it: 1 and itself. For example, the smallest prime number is 2, because only 1 and 2 evenly go into it. And, although it may seem that the number 1 itself may be a prime number, it is not.

The number of prime numbers is actually infinite. The list goes on forever! However, an ancient Greek mathematician, Eratosthenes, developed a way to make a list of at least the small prime numbers, called the Sieve of Eratosthenes.

Objectives

Review these objectives. When you have completed this section, you should be able to:

- Determine whether a number is prime or composite.
- Use divisibility rules to find the prime factorization of a number.
- Express a number as a product of prime numbers.
- List all the factors of a number.
- Find the GCF of two numbers.
- Use divisibility rules to find factors of a number.
- Use a fraction to show part of a whole.
- Represent a fraction on the number line.
- Identify and find equivalent fractions.
- Reduce fractions to lowest terms.

Vocabulary

composite number. A number that has more factors than just 1 and itself.

denominator. The number under the fraction line; tells how many equal parts the whole was broken into.

equivalent fractions. Fractions with the same numerical value; fractions that are equal to each other.

factor. A number that divides evenly into another number.

factor tree. An organized way of finding the prime factorization of a number.

fraction. A number that shows part of a whole.

fraction bar. The line between the numerator and the denominator of a fraction.

greatest common factor. The largest factor that any given numbers have in common.

numerator. The number above the fraction line; tells how many parts of the whole exist.

prime factorization. The product of prime factors of a number.

prime number. A number that has only two factors: 1 and itself.

proper fraction. A fraction in which the numerator is smaller than the denominator.

simplest form. A fraction in lowest terms.

Note: All vocabulary words in this LIFEPAK appear in **boldface** print the first time they are used. If you are not sure of the meaning when you are reading, study the definitions given.

PRIME OR COMPOSITE?

The prime numbers less than 50 include 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47. All of these numbers have only two numbers that evenly go into them (factors): the number

1 and the numbers themselves. Numbers that have more than two factors are called **composite numbers**. For example, the smallest composite number is 4. Its factors are 1, 2, and 4.

Example:

Is 14 prime or composite?

Solution:

To identify whether 14 is prime or composite, check to see if it has more than two factors. The numbers that go into 14 evenly are 1, 2, 7, and 14. So, it is a composite number.

Think about it!

What about the number 1? Is it **prime** or **composite**? It's actually neither! To be prime, a number has to have two factors, and to be composite it has to have at least three factors. Since 1 has only one factor (itself), it is neither prime nor composite.

DIVISIBILITY RULES

In this last example, it was easy for us to determine that 14 is composite. Since it's such a small number, we just know that 2 and 7 go into it. What about larger numbers, though? For example, how could we determine whether the number 351 is prime or composite? We could start trying to divide 351 by random numbers, but it would probably take a while to find a number that would divide evenly. A better way is to use the divisibility rules to guide us. Divisibility rules help determine if a number is divisible by another number. Take a look.

Rule: Any even number is divisible by 2.

So, any whole number that ends in 0, 2, 4, 6, or 8 is divisible by 2. Some examples are 28, 86, 100, and 504.

Rule: When the sum of all the digits in a number is divisible by 3, then the number is divisible by 3.

For this rule, try adding the digits of a number. If that sum is divisible by 3, then the original number is also divisible by 3. For example, 525 is divisible by 3 because $5 + 2 + 5$, or 12, is divisible by 3.

Rule: Any whole number that has a 0 or 5 in the ones place is divisible by 5.

Some examples of numbers that are divisible by 5 are 95, 180, and 3,455.

Rule: Double the digit in the ones place and subtract it from the rest of the number. If that difference is divisible by 7 (including 0), then the original number is also divisible by 7.

Try this test for the number 182. The ones digit is a 2. So, double it and then subtract it from the rest of the number, which is 18.

$$2 \times 2 = 4$$

$$18 - 4 = 14$$

Since 14 is divisible by 7, the original number 182 is also divisible by 7.

For example...

Notice that the sum of the digits of 351 is $3 + 5 + 1$, or 9. Since 9 is divisible by 3, 351 is also divisible by 3. That means that 3 is a factor of 351, since it goes evenly into it. So, 351 is a composite number.

Example:

Show that 147 is divisible by 7.

Solution:

The ones digit is 7. Double it and subtract it from the rest of the number, which is 14.

$$7 \times 2 = 14$$

$$14 - 14 = 0$$

Notice in the rule that zero is considered divisible by 7. So, 147 is divisible by 7.

Let's look at one more rule.

Rule: Alternately subtract and add the digits of a number from left to right. If the result is divisible by 11 (including 0), then the original number is also divisible by 11.

Try this test for the number 8,371. Alternate subtracting and adding the digits from left to right.

$$8 - 3 + 7 - 1$$

$$5 + 7 - 1$$

$$12 - 1$$

$$11$$

Since 11 is divisible by 11, the original number 8,371 is also divisible by 11.

Example:

Show that 4,004 is divisible by 11.

Solution:

Alternate subtracting and adding the digits from left to right.

$$4 - 0 + 0 - 4$$

$$4 + 0 - 4$$

$$4 - 4$$

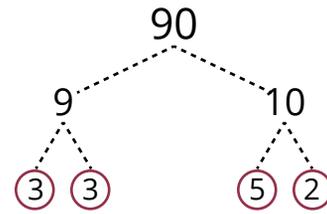
$$0$$

Since 0 is divisible by 11, the original number 4,004 is also divisible by 11.

PRIME FACTORIZATION

The divisibility rules are useful for determining if a prime number is a factor of another number. Determining the prime factors of a number will be useful to us in our work with fractions later on. In fact, we'll want to express a number as a product of prime numbers. This is called a number's **prime factorization**. One method for finding the prime factorization of a number is to use a **factor tree**, pictured on the right.

As you can see, the prime factorization of 90 is $2 \times 3 \times 3 \times 5$. Notice that the factors are listed in order from smallest to largest. Notice, also, that there is a repeated factor. Remember that



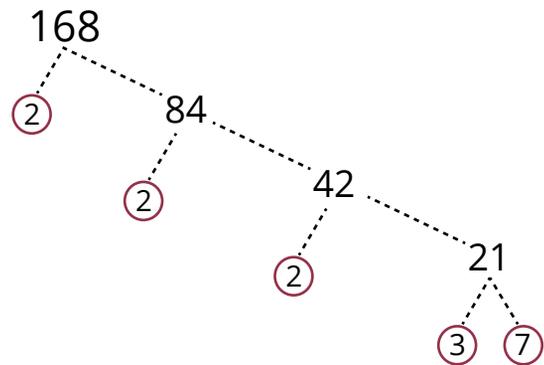
$$90 = 2 \times 3 \times 3 \times 5$$

repeated multiplication can be represented using exponents. Since there are two factors of 3, they can be expressed as 3^2 . So, the prime factorization of 90 is written as $2 \times 3^2 \times 5$. Look at one more example.

Example:

Find the prime factorization of 168.

Solution:



$$168 = 2^3 \times 3 \times 7$$

Let's Review!

Before going on to the practice problems, make sure you understand the main points of this lesson.

- ✓ A prime number has only two factors: 1 and itself. A composite number has at least three factors.
- ✓ The divisibility rules are helpful in determining if one number goes into another number.
- ✓ Every number can be expressed as a product of prime numbers, called its prime factorization.
- ✓ Factor trees are useful for finding the prime factorization of a number.

Place a check mark next to each correct answer (you may select more than one answer).

1.10 Use the divisibility rules to determine which of the following numbers 504 is divisible by.

- 2
- 3
- 5
- 7
- 11

1.11 Use the divisibility rules to determine which of the following numbers 330 is divisible by.

- 2
- 3
- 5
- 7
- 11

1.12 Use the divisibility rules to determine which of the following numbers 924 is divisible by.

- 2
- 3
- 5
- 7
- 11

1.13 Use the divisibility rules to determine which of the following numbers 315 is divisible by.

- 2
- 3
- 5
- 7
- 11

GREATEST COMMON FACTOR

Do you remember what a factor is? It's a number that goes into another number. For example, 2 is a factor of 10 because 2 goes into 10. The divisibility rules are helpful for determining if a number is a factor.

In this lesson, we'll use the divisibility rules to help us as we find all the factors of a number and the **greatest common factor** of two numbers.

MORE DIVISIBILITY RULES

The divisibility rules are helpful for finding some of the prime factors of a number. Let's review those rules again.

- A number is divisible by 2 if the digit in the ones place is a 2, 4, 6, 8, or 0.
- A number is divisible by 3 if the sum of the digits is divisible by 3.
- A number is divisible by 5 if the digit in the ones place is a 5 or 0.
- Subtract twice the digit in the ones place from the rest of the number. If this difference is divisible by 7, then the number is divisible by 7.
- Alternately subtract and add the digits. If this result is divisible by 11, then the number is divisible by 11.

Here are a few more divisibility rules that can be used to find some of the composite factors of a number.

Rule: If the last two digits of a number are divisible by 4, then the entire number is divisible by 4.

Rule: If the number is divisible by both 2 and 3, then it is divisible by 6.

Rule: If the sum of all the digits in a number is divisible by 9, then the number is divisible by 9.

Rule: Any whole number that has a 0 in the ones place is divisible by 10.

This might help!

Remember that a prime number is a number that has exactly two factors: 1 and itself. A composite number is a number that has more than two factors.

Example:

Check if 324 is divisible by 4, 6, 9, or 10.

Solution:

Let's see which rules apply to the number.

- 4: Check if the last two digits are divisible by 4. The last two digits of 324 are 24. Since 24 is divisible by 4, the whole number is divisible by 4.
- 6: Check if the number is divisible by 2 and 3. The number 324 is even (ends in 2, 4, 6, 8, or 0), so it's divisible by 2. The sum of the digits is $3 + 2 + 4$, or 9, which is divisible by 3. So, 324 is divisible by 3. Since 324 is divisible by both 2 and 3, it is divisible by 6.
- 9: Check if the sum of the digits is divisible by 9. The sum of the digits is $3 + 2 + 4$, or 9, which is divisible by 9. So, 324 is divisible by 9.
- 10: Check if the ones digit is a 0. In 324, the digit in the ones place is a 4, so it's not divisible by 10.

The number 324 is divisible by 4, 6, and 9.

Did you know?

There are two numbers that are a factor of every number: 1 and itself.

FINDING FACTORS

All of these divisibility rules can be helpful in finding the factors of a whole number. Keeping in mind these rules, think of pairs of numbers that multiply to get a number. For example, let's find all the factors of 45. The first pair of numbers that multiply to get 45 is 1 and 45. Since 45 is not an even number, it isn't divisible

by 2 (or 4, 6, and 10). However, the sum of the digits is $4 + 5$, or 9, which is divisible by both 3 and 9. That leads us to two more pairs of numbers that multiply to get 45: 3 and 15, and 5 and 9. So, the factors of 45 are 1, 3, 5, 9, 15, and 45.

Example:

Find all the factors of 48.

Solution:

List the pairs of numbers that multiply to get 48. Use the divisibility rules to help you.

1 and 48
2 and 24
3 and 16
4 and 12
6 and 8

So, the factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.

This might help!

Notice that when we're listing pairs of numbers, we only have to list each pair once. That's because multiplication is commutative. The pair 6 and 8 is the same as 8 and 6. Once the list begins to repeat itself, we can stop looking for pairs.

Factors will be very useful when we start looking at fractions. When we work with fractions, we'll need to be able to look at two or three numbers and find the factors that they have in common. In fact, we'll want to find the largest factor that they have in common. This is called the greatest common factor, or GCF, of two or more numbers. Let's try finding the GCF of 45 and 48, since earlier we made lists of all the factors for each number.

45: 1, 3, 5, 9, 15, 45

48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

GREATEST COMMON FACTOR

The only two factors that 45 and 48 have in common are 1 and 3. So, the greatest common factor, or the largest common factor, of the two numbers is 3. Making lists of all the factors for each number is a great way to find the GCF of two numbers, but there's another great method that uses prime factorization. This method is especially useful for larger numbers that may have a lot of factors. Take a look.

Example:

Find the GCF of 12 and 32.

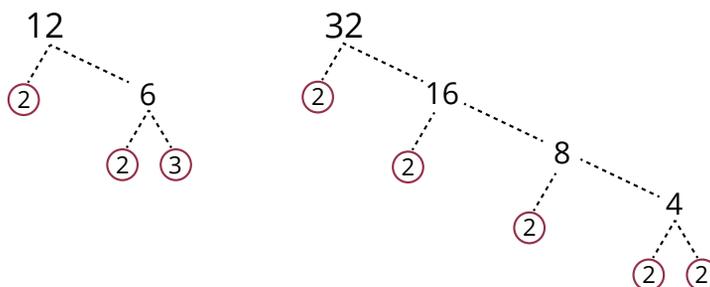
Solution:

First, find the prime factorization for each number.

$$12 = 2 \times 2 \times 3$$

$$32 = 2 \times 2 \times 2 \times 2 \times 2$$

Now, find the common prime factors and multiply them. 12 and 32 have two 2's in common. So, the GCF is 2×2 , or 4.

**Step by Step**

To find the GCF of two numbers, first find the prime factorization of each number. Then, find the common prime factors and multiply them together.

Example:

Find the GCF of 5 and 12.

Solution:

First, find the prime factorization for each number. Since 5 is a prime number, its prime factorization is just 5. We found the prime factorization for 12 in the last example.

$$5: 5$$

$$12: 2 \times 2 \times 3$$

These two numbers have no prime factors in common. That means their greatest common factor is 1, because 1 is the only number that goes into both of them.

Key point!

When two numbers have no common factors, their GCF is 1.

Let's Review!

Before going on to the practice problems, make sure you understand the main points of this lesson.

- ✓ Factors are numbers that evenly go into other numbers.
- ✓ The rules of divisibility can be helpful for finding all the factors of a number.
- ✓ The greatest common factor, or GCF, of two numbers is the largest factor that they have in common.
- ✓ The GCF of two numbers that have no common factors is 1.



Place a check mark next to each correct answer (you may select more than one answer).

- 1.14** Which of the following statements are true of the greatest common factor of two numbers?
- Each number must share the factor.
 - The number must be large.
 - The greatest common factor could be 1.
 - The number must be the largest factor both numbers share.
- 1.15** Use the divisibility rules to determine which of the following numbers 528 is divisible by.
- | | | | | |
|----------------------------|----------------------------|----------------------------|-----------------------------|-----------------------------|
| <input type="checkbox"/> 2 | <input type="checkbox"/> 4 | <input type="checkbox"/> 6 | <input type="checkbox"/> 9 | <input type="checkbox"/> 11 |
| <input type="checkbox"/> 3 | <input type="checkbox"/> 5 | <input type="checkbox"/> 7 | <input type="checkbox"/> 10 | |
- 1.16** Choose all the factors of 5.
- | | | | | |
|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|
| <input type="checkbox"/> 1 | <input type="checkbox"/> 2 | <input type="checkbox"/> 3 | <input type="checkbox"/> 4 | <input type="checkbox"/> 5 |
|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|
- 1.17** Choose all the factors of 12.
- | | | | |
|----------------------------|----------------------------|----------------------------|-----------------------------|
| <input type="checkbox"/> 1 | <input type="checkbox"/> 4 | <input type="checkbox"/> 7 | <input type="checkbox"/> 10 |
| <input type="checkbox"/> 2 | <input type="checkbox"/> 5 | <input type="checkbox"/> 8 | <input type="checkbox"/> 11 |
| <input type="checkbox"/> 3 | <input type="checkbox"/> 6 | <input type="checkbox"/> 9 | <input type="checkbox"/> 12 |
- 1.18** Choose all the factors of 8.
- | | | | |
|----------------------------|----------------------------|----------------------------|----------------------------|
| <input type="checkbox"/> 1 | <input type="checkbox"/> 3 | <input type="checkbox"/> 5 | <input type="checkbox"/> 7 |
| <input type="checkbox"/> 2 | <input type="checkbox"/> 4 | <input type="checkbox"/> 6 | <input type="checkbox"/> 8 |

Answer true or false.

- 1.19** _____ The list of all factors for 21 is 1, 3, 7, and 21.
- 1.20** _____ The list of all factors for 32 is 1, 2, 4, 8, and 32.

Circle the letter of each correct answer.

- 1.21** Find the GCF of 14 and 22.
- | | | | |
|------|------|------|------|
| a. 1 | b. 2 | c. 4 | d. 7 |
|------|------|------|------|
- 1.22** Find the GCF of 20 and 30.
- | | | | |
|------|------|-------|-------|
| a. 2 | b. 5 | c. 10 | d. 15 |
|------|------|-------|-------|
- 1.23** Find the GCF of 16 and 27.
- | | | | |
|------|------|------|------|
| a. 1 | b. 2 | c. 3 | d. 8 |
|------|------|------|------|

Write the correct answer on each blank

- 1.24** The GCF of 18 and 36 is _____ .
- 1.25** The GCF of 44 and 66 is _____ .
- 1.26** The GCF of 15 and 35 is _____ .

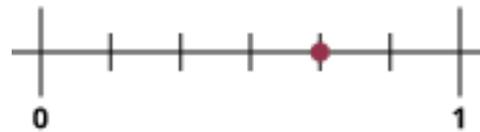
SELF TEST 1: FACTORS AND FRACTIONS

Answer true or false (each answer, 7 points).

- 1.01 _____ If the sum of the digits of a number is divisible by 2, then the original number is also divisible by 2.
- 1.02 _____ The fraction $\frac{9}{7}$ is *not* a proper fraction.

Circle each correct answer (each answer, 6 points).

- 1.03 In the fraction $\frac{7}{8}$, the _____ is 7.
 a. numerator
 b. denominator
- 1.04 The number 37 is
 a. composite
 b. prime
- 1.05 What is the prime factorization of 48?
 a. 4×12
 b. $2^3 \times 6$
 c. $2^4 \times 3$
 d. $2^3 \times 3^2$
- 1.06 Which fraction is modeled in the circle?
 a. $\frac{3}{8}$
 b. $\frac{3}{6}$
 c. $\frac{4}{8}$
 d. $\frac{2}{6}$
- 1.07 What is the value of the point on the number line?
 a. $\frac{3}{8}$
 b. $\frac{4}{6}$
 c. $\frac{5}{8}$
 d. $\frac{5}{7}$
- 1.08 In history class, students could either make a timeline of the first Gulf War or interview someone that lived through the first Gulf War. Nine of the students chose to make a timeline, and eleven of the students chose to interview someone. Which fraction represents the part of the class that interviewed someone?
 a. $\frac{9}{20}$
 b. $\frac{1}{11}$
 c. $\frac{9}{11}$
 d. $\frac{11}{20}$
- 1.09 Which two fractions are *not* equivalent?
 a. $\frac{7}{8}, \frac{5}{6}$
 b. $\frac{1}{9}, \frac{2}{18}$
 c. $\frac{3}{16}, \frac{9}{48}$
 d. $\frac{6}{15}, \frac{4}{10}$



Place a check mark next to each correct answer (you may select more than one answer; each question, 8 points).

1.010 Choose all the numbers that 810 is divisible by.

- | | | |
|----------------------------|----------------------------|-----------------------------|
| <input type="checkbox"/> 2 | <input type="checkbox"/> 5 | <input type="checkbox"/> 9 |
| <input type="checkbox"/> 3 | <input type="checkbox"/> 6 | <input type="checkbox"/> 10 |
| <input type="checkbox"/> 4 | <input type="checkbox"/> 7 | <input type="checkbox"/> 11 |

1.011 Choose all the numbers that are factors of 16.

- | | | | | |
|----------------------------|----------------------------|----------------------------|-----------------------------|-----------------------------|
| <input type="checkbox"/> 1 | <input type="checkbox"/> 3 | <input type="checkbox"/> 5 | <input type="checkbox"/> 8 | <input type="checkbox"/> 12 |
| <input type="checkbox"/> 2 | <input type="checkbox"/> 4 | <input type="checkbox"/> 6 | <input type="checkbox"/> 10 | <input type="checkbox"/> 16 |

Fill in each blank with the correct answer (each question, 7 points).

1.012 The GCF of 28 and 30 is _____.

1.013 The GCF of 14 and 15 is _____.

1.014 Write $\frac{18}{42}$ in simplest form.

_____ / _____

1.015 Write a fraction that is equivalent to $\frac{3}{4}$.

_____ / _____

	<p>SCORE _____</p>	<p>TEACHER _____</p> <p style="font-size: small; text-align: right;">initials date</p>
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